Estimating Discount Functions with Consumption Choices over the Lifecycle*

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Abstract

We estimate β-δ time preferences and relative risk aversion (RRA) using a lifecycle model including stochastic income, liquid and illiquid assets, credit cards, dependents, Social Security, mortality, and bequests. Preference parameters are identified by cross-tabulating four lifecycle age intervals and four balance sheet moments: the proportion of households carrying (i.e., revolving) credit card debt, average carried credit card debt, average net wealth among households carrying credit card debt, and average net wealth among households not carrying credit card debt. The sixteen moments are approximately matched by (MSM) parameter estimates $\beta = 0.53$, $\delta = 0.99$, and $RRA = 1.9$.

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1 Introduction

Economic models predict that intertemporal preferences influence most important decisions, including human capital investment, work effort, nutritional choices, exercise, medical adherence, smoking, alcohol consumption, saving, and borrowing.

Most research on time preferences uses lab or field experiments in which the experimenter asks participants to make a series of choices between sooner, smaller rewards and later, larger rewards.[1] Experiments have the benefit of giving the researcher control over the options that are offered to the decision-maker (e.g., Augenblick et al., 2015).

The current paper sacrifices that experimental control in return for the benefit of estimating time preferences using lifecycle consumption/saving/borrowing choices that, in aggregate, generate large-stakes consequences. We use a lifecycle model to structurally estimate time preferences by analyzing the evolution of four balance sheet moments: the proportion of households borrowing on their credit cards (i.e., holding some high-interest credit card debt that revolves from one month to the next), the average magnitude of credit card borrowing (i.e., the average magnitude of high-interest credit card debt that revolves from one month to the next, including zeroes in the average), total net wealth among households revolving credit card debt, and total net wealth among households not revolving credit card debt. We measure each of these four balance sheet moments in four age intervals: 21-30, 31-40, 41-50, and 51-60. Given a stochastic income process, these lifecycle balance sheet choices are the dual of the stochastic lifecycle consumption path. Our benchmark estimates study households with a completed high school degree and no college degree, as this is the largest educational category for the period we study.[2]

Our empirical moments, calculated from the Survey of Consumer Finances, suggest that

[1] For reviews, see e.g., Ainslie (1992), Frederick et al. (2002), Ericson and Laibson (2019), and Cohen et al. (2020).

[2] Results for other educational groups are reported in our appendix.

households can appear to be either impatient or patient, depending on what part of the balance sheet the viewer is studying. On the one hand, we observe a high rate of credit card rollover borrowing (e.g., the majority of households aged 21 through 60 revolve a credit card balance). On the other hand, we also observe that households accumulate sizable amounts of (mostly illiquid) wealth over the lifecycle (e.g., even conditional on revolving a credit card balance, households aged 51-60 *simultaneously* hold average wealth of 4.6-times their average income). The observed rates and magnitudes of credit card borrowing and wealth formation may or may not pose a conceptual tension for classical economic models of household behavior. For example, some (classical) households will experience a sequence of shocks that lead to credit card borrowing that coincides with a large stock of (previously accumulated) illiquid wealth. To evaluate whether the population-level distribution of balance sheet data poses a quantitative tension, we use a calibrated structural model.

Lifecycle balance sheet choices depend complexly on financial constraints, preferences, and behavioral biases (Gomes et al., 2021). To map field data to fundamental preference parameters, the structural lifecycle model that we use is designed to reflect key features of the institutional environment in which the field data was generated (DellaVigna, 2019). We solve and simulate a model of lifecycle consumption and saving/borrowing choices, which builds on the "buffer stock" savings literature (e.g., Deaton, 1991; Carroll, 1992, 1997; Angeletos et al., 2001; Gourinchas and Parker, 2002; Kaplan and Violante, 2014; Kaplan et al., 2018; Choukhmane, 2019). Our model features liquid and illiquid assets, credit card debt, liquidity constraints, age-varying dependents (both children and older adults), Social Security, stochastic income, mortality, and bequests.

Our central departure from classical economic models is that we allow for a more flexible specification of time preferences. A large body of empirical research finds that a substantial fraction of economic agents have *present-focused preferences*, meaning that "agents are more likely in the present to choose an action that generates immediate experienced utility, than they would be if all the consequences of the actions in their choice set were delayed by the same amount of time. More informally, this amounts to people choosing more impatiently for the present than they do for the future" (Ericson and Laibson, 2019, p. 3).

There are many ways to represent time preferences in general and present-focused preferences in particular. We use the two-parameter discount function introduced by Phelps and Pollak (1968) to study inter-generational discounting and then used by Laibson (1997) to study intra-personal discounting: $1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots$. This discount function can be calibrated to generate exponential discounting ($\beta = 1$) or a particular form of present-focused preferences that is referred to as *present bias*: i.e., $\beta < 1$. When $\beta < 1$, the discount rate is greater in the short-run than the long-run.[3] Accordingly, by generalizing the discount function with present bias we allow for households' time preferences to reflect the potential tension between acting impatiently and acting patiently that is exhibited by our empirical moments.

Adopting the methodology of Gourinchas and Parker (2002), we use a two-stage Method of Simulated Moments (MSM) procedure to estimate three preference parameters in our model: β and δ (the time preference parameters), as well as a (constant) coefficient of relative risk aversion (RRA).[4] Our parameter estimates minimize a weighted loss function that compares the 16 empirical moments from U.S. household balance sheets to the same moments generated by simulating the structural model.

When we estimate an exponential discount function (i.e., constraining $\beta = 1$), the MSM procedure estimates an annual discount factor of $\hat{\delta} = 0.96$ and $\widehat{RRA} = 1.5$. When we estimate a present-biased discount function, the MSM procedure jointly estimates $\hat{\beta} = 0.53$,

[3] Our main specifications assume that households are naive, meaning that they do not anticipate the self-control problems of future selves (Strotz, 1956; Akerlof, 1991; O'Donoghue and Rabin, 1999a,b). In most discrete-time consumption models, sophisticates and naifs behave similarly (Angeletos et al., 2001). In a model like ours, Maxted (2022) uses continuous-time methods to prove an observational equivalence between the consumption choices of sophisticates versus naifs. Section 7 extends our model to the case of sophistication, and the results are similar.

[4] See McFadden (1989), Pakes and Pollard (1989), and Duffie and Singleton (1993) for early references in the MSM literature.

$\hat{\delta} = 0.99$, and $\widehat{RRA} = 1.9$. The standard error on $\hat{\beta}$ in the present-bias specification is 0.11. Using a 1% significance threshold, our estimate of $\hat{\beta}$ rejects the null hypothesis of exponential discounting ($\beta = 1$).

Together, our estimates of $\hat{\beta}$ and $\hat{\delta}$ allow our lifecycle model to match the tension between acting impatiently versus patiently that we observe in households' financial choices. Over the coming year, the *short-run* discount rate is approximated by $1 - \hat{\beta}\hat{\delta} = 48\%$. The high short-run estimated discount rate implies that households are likely to borrow on credit cards at high real interest rates; the real interest rate on credit cards is 11% in our benchmark calibration. This frequent and substantial borrowing at an 11% interest rate – consistent with our empirical credit card borrowing moments – makes households appear to be impatient.

For all subsequent years, the continuation discount rate is only $1 - \hat{\delta} = 1\%$ per year. This low *long-run* estimated discount rate implies that households are willing to accumulate substantial (illiquid) retirement wealth at a calibrated real interest rate of 5%.[5] Intuitively, when times are occasionally very good (e.g., a large positive transitory income shock), households put their extra liquidity into the illiquid asset (which pays a small premium over the liquid asset). Households use the illiquid asset to store wealth whether they are naive (our benchmark case) or sophisticated (an appendix case). Naive households put their extra funds in the illiquid asset because they anticipate that they will not need the liquidity in the medium run. On the other hand, sophisticated households are partially motivated to put their extra funds in the illiquid asset as a commitment device. Hence, whether households are naive or sophisticated they will hold substantial illiquid wealth and, nevertheless, maintain minimal liquidity buffers most of the time.[6]

Under present bias, the majority of simulated households borrow on credit cards in

[5]This is the calibrated historical real return on illiquid assets like housing. For example, Kaplan and Violante (2014) calibrate the long-run real return on housing to be 6.29%.

[6]In addition to fitting our empirical moments, these patterns are also consistent with empirical findings documented in other papers, notably Angeletos et al. (2001), Laibson et al. (2003), Kaplan and Violante (2014), Kaplan et al. (2018), Laibson et al. (2021a), and Lee and Maxted (2023).

any given period of working life, consistent with our empirical evidence. At an 11% real interest rate on credit card borrowing and a 5% real return on illiquid investment, *exponential discounting* cannot explain why most households roll over credit card debt and simultaneously accumulate high levels of illiquid wealth. However, *if* we assume different interest rates, most importantly, a lower real interest rate on credit card borrowing (e.g., 6% rather than our assumed value of 11%), the exponential model does match the balance sheet moments. Using a lower credit card borrowing rate we estimate $\hat{\beta}$ to be close to one. By varying our interest rate assumptions in our robustness analysis, we demonstrate the relationship between our model and exponential discounting models that assume different interest rates than we do (e.g., Kaplan and Violante, 2014; Kaplan et al., 2018). Accordingly, our simulation shows that frequent high-interest-rate borrowing is explained by present bias, whereas frequent low-interest-rate borrowing is consistent with exponential discounting.

A potential downside of structural modeling with complex household preferences, budget constraints, and state spaces, is that it is often difficult to understand the economic mechanisms driving parameter identification. We propose a novel identification analysis, which we refer to as *boundary analysis*, that explains how our structural lifecycle model identifies β, δ, and RRA. This graphical analysis draws boundaries around points in the (β, δ, RRA) parameter space that fit selected subsets of the household balance sheet moments that the model is attempting to match. In doing so, our boundary analysis provides a visualization of the parameter subspaces that are consistent with moment matching. For our benchmark calibration (with an 11% real interest rate on credit cards), the parameter subspace that fits the moments does not include any mass in the set of exponential discounting models (i.e., $\beta = 1$). The exponential model can fit the wealth moments *or* the credit card moments, but not both.[7]

[7] However, as noted above, the exponential model can fit all moments if we substantially lower the real interest rate on credit cards (or raise the return on illiquid assets). With a credit card interest rate that is close to the rate of return on the illiquid asset, the exponential discounting model can explain why most

Our parameter estimates are broadly consistent with other papers that have estimated present-biased time preferences with structural models, particularly when field data is collected from households with relatively lower income. Paserman (2008) obtains identification from heterogeneity in unemployment durations and reservation wages to estimate $\beta = 0.40$ for workers with low income and $\beta = 0.89$ for workers with high income. DellaVigna et al. (2017) also use a model of unemployment durations to estimate $\beta = 0.58$. Using data on welfare recipients, Fang and Silverman (2009) estimate $\beta = 0.36$. Skiba and Tobacman (2018) model payday loan borrowing to estimate $\beta = 0.50$. Jones and Mahajan (2015) run a field experiment with income tax refunds for households with low income and estimate $\beta = 0.34$. Ganong and Noel (2019) measure the trajectory of consumption among unemployed households, using consumption dynamics at the time of unemployment insurance exhaustion as a key source of identification of time preferences. They estimate that one-quarter of households have $\beta = 0.5$, and the rest have $\beta = 0.9$. Lockwood's (2020) Figure 2 summarizes the literature and reports a value of $\beta = 0.5$ for households with low income and 0.9 for households with high income.

Some papers with structural models and field data estimate higher levels of β, even for populations with low income. Allcott et al. (2022) estimate β between 0.74 and 0.83 for payday loan borrowers. Martinez et al. (2022) use the timing of tax filing data for low income households to estimate $\beta = 0.86$ in their preferred specification.

There is a much larger literature that estimates β using laboratory/experimental evidence, which is reviewed in Ericson and Laibson (2019) and Cohen et al. (2020). Relative to the literature using structural models and field data, the literature using experimental evidence tends to find higher estimates of β.[8]

households are simultaneously borrowing on credit cards and accumulating large stocks of partially illiquid wealth.

[8]For example, Augenblick et al. (2015) study college students and estimate $\beta = 0.97$ for money-earlier-or-later (MEL) choices and $\beta = 0.89$ for a time-stamped, effort-based task. Using similar time-stamped, effort-based experimental paradigms, Augenblick and Rabin (2019) and Fedyk (2022) also study college

2 Wealth and Credit Card Data

We begin by empirically documenting household (net) wealth accumulation and credit card borrowing. Together these moments characterize the tension between acting patiently (i.e., saving for the long run) and acting impatiently (i.e., revolving credit card debt). We will use these moments in our structural model to estimate the discount function parameters that are needed to replicate the household balance sheet patterns that we observe in the data.

To capture households' financial behavior over the lifecycle, we estimate moments separately for households with heads aged 21-30, 31-40, 41-50, and 51-60.[9] All empirical moments are calculated using the Survey of Consumer Finances (SCF) from 1995 through 2013. Our analysis focuses on U.S. households whose head has a high school degree and no college degree (this group constitutes 57% of SCF households from 1995 through 2013).[10] Our moment estimates control for household demographics, business cycle effects, and cohort effects in order to make the characteristics of the SCF population analogous to our simulated data. Appendix A contains a detailed description of the estimation procedures.

Before presenting our moment estimates, one important point to emphasize is that we restrict the SCF sample to households that *possess* a credit card. Relative to the raw SCF, this will increase our estimates of the share of households with revolving credit card debt. We make this restriction to align the SCF population with our model's simulated population, since our model provides households with a credit card.[11] There are clear benefits to possessing a credit card, and simply using a credit card does not imply that a household must revolve an interest-bearing balance on that card. However, supply-side restrictions

students and estimate $\beta = 0.83$ and $\beta = 0.82$, respectively. Goda et al. (2019) study a representative sample and use a MEL design to elicit time preferences; they find that the *average* β is close to one, and that variation in β across respondents strongly predicts the level of retirement savings.

[9]We view post-retirement financial decisions as sufficiently interesting that they call for specialized models, beyond the scope of what we study here, in order to realistically incorporate health shocks and intergenerational transfers (e.g., De Nardi et al., 2010; Ameriks et al., 2020).

[10]We discuss results for other education groups in Section 7.

[11]See also Lee and Maxted (2023) for further discussion of this restriction.

often inhibit financially fragile households from accessing credit cards (e.g., Bornstein and Indarte, 2023). So, by restricting the SCF sample to households that have a credit card, we generate our empirical measurements with the types of households that we model (i.e., households that have enough financial strength to obtain a credit card in the first place).[12]

Our moment estimates are summarized in Table 1, and are described below. In Table 1 we adopt the notation $\bar{m}_{J_m}$ for the vector of moments and $se\left(\bar{m}_{J_m}\right)$ for their standard errors, consistent with the notation in Section 4 below.

Table 1: Second-Stage Moments

Name	$\bar{m}_{J_m}$	$se\left(\bar{m}_{J_m}\right)$
% Visa:		
21-30	0.640	0.021
31-40	0.629	0.026
41-50	0.588	0.029
51-60	0.503	0.037
mean Visa:		
21-30	0.111	0.012
31-40	0.096	0.014
41-50	0.110	0.017
51-60	0.104	0.020
wealth | debt:		
21-30	1.222	0.117
31-40	1.868	0.167
41-50	3.377	0.227
51-60	4.650	0.340
wealth | no debt:		
21-30	1.659	0.129
31-40	2.800	0.154
41-50	4.613	0.252
51-60	8.071	0.393

Source: Authors' estimation based on data from the Survey of Consumer Finances. Note: Estimates pertain to households with heads who have a high school diploma and no college degree. Standard errors are calculated using the procedure outlined in the 2013 SCF Codebook, and account for both imputation error and sampling error. See Appendix A for estimation details.

[12] Lee and Maxted (2023) also show that households without credit cards often substitute to higher-cost credit margins such as payday loans, though these nonbank margins are not well captured by the SCF (Zinman, 2015).

The first statistic, $\%Visa$, is the fraction of households that borrow on credit cards.[13] For age groups [21-30, 31-40, 41-50, 51-60], we find that respectively [64.0%, 62.9%, 58.8%, 50.3%] of households carry high-interest credit card debt from one month to the next. Specifically, $\%Visa$ represents the fraction of households that self-report that they did not pay their bill in full at the end of the last billing cycle. Throughout, we restrict our measure of credit card debt to balances on which the household reports paying an interest rate above 5% in order to exclude promotional low-APR products, following Lee and Maxted (2023).

We construct the second statistic, $meanVisa$, by dividing the quantity of high-interest credit card debt (including zeros for households without such debt) by mean age-specific income. We then average this fraction over the decadal age bins. For age groups [21-30, 31-40, 41-50, 51-60] the average household has outstanding credit card debt equal, respectively, to [11.1%, 9.6%, 11.0%, 10.4%] of the mean income of its age cohort. This statistic is effectively a ratio of means. We use the ratio of means instead of the mean ratio since household-level income can take small values.[14,15]

The third statistic, $wealth|debt$, is also computed as a ratio of means. We divide average net worth among households that did not pay off their credit card debt in full at the end of the last billing cycle by mean age-specific income.[16] For households with heads aged [21-30, 31-40, 41-50, 51-60], respectively, the resulting $wealth|debt$ measures equal [1.22, 1.87, 3.38,

[13]This is the fraction that borrows on any type of card, not just Visa cards.

[14]Compared to the Federal Reserve's G19 Consumer Credit Series and the Consumer Credit Panel of the New York Fed/Equifax, the SCF survey questions on credit card borrowing involve substantial underreporting of aggregate borrowing (Zinman, 2009). We adjust the SCF magnitudes upwards to account for underreporting. See Appendix A for details, and Table 4 for robustness to moment choices.

[15]Both of our credit card moments focus only on high-interest credit card debt, not on net liquid wealth. As a consequence, many of the households that are included in our $\%Visa$ and $meanVisa$ moments may still hold some stock of liquid assets simultaneously (i.e., the well-known "co-holding puzzle"). Following similar arguments as in Lee and Maxted (2023), we focus on the debt side of household balance sheets because households' usage of revolving credit card debt reveals their marginal intertemporal price of consumption (even if they are also co-holding). As we discuss in Section 3 below, however, our model will not allow for the co-holding of credit card debt and *liquid* assets, and hence we cannot rule out that many of the factors believed to underly the co-holding puzzle (outlined e.g. in Gomes et al., 2021) may also be partially at play in explaining the borrowing patterns we document.

[16]Most household wealth is held in illiquid assets (whether or not they have credit card debt). Over 80% of net wealth in our SCF sample is held in home equity, defined-contribution retirement accounts, and vehicles.

4.65]. Importantly, these $wealth|debt$ moments show that wealth accumulation is substantial, even for households who are simultaneously borrowing on their credit cards.

The final statistic, $wealth|no\ debt$, is similar to $wealth|debt$ but conditions on having no outstanding credit card balance. Households without credit card debt hold more wealth than households with credit card debt. For households with heads aged [21-30, 31-40, 41-50, 51-60] we find $wealth|no\ debt$ equal to [1.66, 2.80, 4.61, 8.07], respectively.

3 Lifecycle Consumption-Saving Model

This section presents the lifecycle model. Our work extends the numerical simulation literature pioneered by Zeldes (1989), Deaton (1991), Carroll (1992, 1997), Hubbard et al. (1995), Gourinchas and Parker (2002), and Cagetti (2003).[17] Our specific analysis is most closely related to that of Gourinchas and Parker (2002), whose two-stage MSM procedure we adopt (details in Section 4).

The first-stage parameters for this structural model can be found in Table 2, and are described here. Appendix B provides additional details on our first-stage parameter estimation procedures. In the model, economic decision-making begins at age 20. Households have an age-dependent survival hazard of s_t calibrated with data from Social Security Administration life tables. Household composition varies deterministically with age as children and adult dependents enter and leave the household.[18] Effective household size n_t equals the number of household heads – which we assume to be two in our benchmark model – plus the number of dependent adults, plus 0.4 times the number of children under 18. Our robustness analysis studies alternate assumptions about the returns to scale in household consumption.

[17] We have commonalities with many recent papers including Becker and Shabani (2010), De Nardi et al. (2010), Bucciol (2012), Kaplan and Violante (2014), Kaplan et al. (2014), Carroll et al. (2017), Pagel (2017), Auclert et al. (2018), Choukhmane (2019), Attanasio et al. (2020), De Nardi et al. (2020), and Lee and Maxted (2023).

[18] Demographic profiles are estimated parametrically using the decennial census in 1980, 1990, and 2000, and the American Community Survey from 2001 through 2014.

Let Y_t represent period t after-tax income from transfers and wages, including labor income, inheritances, private defined-benefit pensions, and government transfers including Social Security. We model $y_t = \ln(Y_t)$ as the sum of a cubic polynomial in age, family-size effects, an AR(1), and an i.i.d. shock. We approximate the AR(1) with a Markov process, and denote the Markov state ζ.[19] The income process is estimated from the Panel Study of Income Dynamics (PSID).

Let X_t represent liquid asset holdings at the beginning of period t, before receipt of income Y_t. If $X_t < 0$ then uncollateralized debt – i.e., credit card debt – was held between $t-1$ and t. Households face a credit limit at age t of λ_t times average income at age t: $X_t \geq -\lambda_t \overline{Y}_t$. λ_t is a quadratic in age that we estimate from the SCF.[20] The model precludes consumers from simultaneously holding liquid assets and credit card debt, though such potentially suboptimal behavior has been documented among a subpopulation of consumers.[21]

Positive liquid asset holdings earn a risk-free real after-tax gross interest rate of R. We calibrate R as the inflation- and tax-adjusted average of AAA corporate bond yields from 1995-2013.[22] The model features a "soft constraint" that drives a wedge between the return on borrowing versus saving. Specifically, households pay a real interest rate on credit card borrowing of R^{CC}. We refer to this simply as the credit card interest rate, but our computation of R^{CC} accounts for the impact of inflation and bankruptcy, which lower consumers' effective interest payments.[23]

[19]Our benchmark specification does not model income using separate processes for the working lifetime and retirement. Since we do not condition the second-stage moments on work status, the data and model are more closely analogous with an income process that is smooth in age. Alternate assumptions are explored in the robustness section.

[20]This is a crude representation of the income-based credit limits that are common in the revolving credit market (Fulford and Schuh, 2015).

[21]For more, see e.g., Gross and Souleles (2002), Telyukova and Wright (2008), Bertaut et al. (2009), Telyukova (2013), and Gorbachev and Luengo-Prado (2019). See also the discussion in footnote 15.

[22]Specifically, we multiply bond yields by 0.75 to account for taxes, and then subtract the corresponding realized inflation rate.

[23]Specifically, from the quarterly credit card interest rates reported in the Federal Reserve Board's G-19 historical series, we subtract the realized inflation and bankruptcy rates. We calculate the latter by dividing the number of bankruptcies (American Bankruptcy Institute) by the number of households that have credit cards (approximated as 65% of households). This attributes all U.S. bankruptcies to households holding

Let Z_t represent (net) illiquid asset holdings at the beginning of period t, with $Z_t \geq 0$ for all t. Illiquid assets include durables, which generate two types of returns: capital gains and consumption flows. For computational tractability, we set net capital gains equal to zero (i.e., $R^Z = 1$) and fix the annual consumption flow at $\gamma Z_t = 0.05 \cdot Z_t$. Hence, the return from holding the illiquid asset is a 5% annual flow of consumption. We adopt the assumption that liquidating Z_t requires a proportional transaction cost, which declines as a logistic with age. Specifically, the proportional cost of liquidation at age t is $\kappa_t = \frac{1/2}{1+e^{(t-50)/10}}$. These choices about Z do not match the properties of any particular illiquid asset, though Z has some features of home equity and some features of defined-contribution pensions.

Two observations motivate our assumptions about the partial, age-dependent illiquidity of Z. First, despite increasing financial innovation, many household assets continue to be partially illiquid. Accessing equity in homes, cars, and retirement plans entails transaction costs and delays. Indeed, some of these frictions are by design: e.g., early withdrawal penalties in defined-contribution plans and (essentially) complete illiquidity of defined-benefit plans.

Second, illiquid assets taken as a whole tend to become more liquid as a household ages. For example, IRS regulations establish that an employee with a 401(k) may only make a hardship withdrawal if (i) the employer allows such withdrawals, (ii) the household faces an immediate and heavy financial need, (iii) the employee couldn't reasonably obtain the funds from another source, and (iv) the withdrawal is limited to the amount necessary to satisfy that financial need.[24] To verify these preconditions, the household goes through a time-consuming application process. Even if this application is approved, the household still pays income taxes and a 10% penalty on the hardship distribution. By contrast, a mid-life worker who has *previous* employers, can roll over 401(k) balances from those previous

credit cards. Section 6.3 presents robustness checks using a wide range of credit card interest rates.

[24] Secure 2.0, signed into law in December 2022, marginally expands the liquidity of 401(k)s.

employers into an IRA, which allows uncapped distributions without hardship restrictions; IRA distributions are still subject to income taxes and a 10% penalty (Beshears et al., 2022). At age $59\frac{1}{2}$ the early withdrawal penalty is eliminated.

Similar trajectories of rising liquidity characterize housing wealth. For example, a new homeowner will not be able to take out a home equity loan if the household has a high loan-to-value ratio. However, an older homeowner is likely to have a lower loan-to-value ratio due to the mechanical process of principal repayment and the probabilistic process of home value appreciation (Liu, 2022). Accordingly, home equity tends to become less and less illiquid (on the margin) as households age. However, home equity loans generate transaction costs and inherent time delays, which may be enough to materially discourage their use by households seeking immediate gratification.

Ultimately, while our modeling of asset illiquidity qualitatively captures certain features of existing institutions, the illiquid Z account is a coarse modeling tool overall. Besides our reduced-form liquidity assumptions, another simplification is that we treat the illiquid account as net wealth, which prevents us from studying households' willingness to utilize secured debt (e.g., mortgages) to purchase illiquid assets. We also assume throughout that households have model-consistent expectations about the return on their illiquid assets (we provide a preliminary analysis of overoptimistic return expectations in Section 6.3). Though our robustness checks will evaluate the sensitivity of our results to the liquidity properties of Z, we view enriching the modeling of illiquid assets as an important task for future work.

Let I_t^X represent net investment into the liquid asset X during period t, and let I_t^Z represent net investment into the illiquid asset Z during period t. The dynamic budget constraints are given by:

$$X_{t+1} = R^X \left(X_t + I_t^X \right), \text{ and} \tag{1}$$

$$Z_{t+1} = R^Z \left(Z_t + I_t^Z \right). \tag{2}$$

Since the interest rate on liquid wealth R^X depends on whether the consumer is borrowing or saving in their liquid account,

$$R^X = \begin{cases} R^{CC} & \text{if} \quad X_t + I_t^X < 0 \\ R & \text{if} \quad X_t + I_t^X \geq 0 \end{cases}.$$

The static budget constraint is:

$$C_t = Y_t - I_t^X - I_t^Z + \kappa_t \min \left(I_t^Z, 0 \right).$$

The state variables at the beginning of period t are age (t), liquid wealth ($X_t + Y_t$), illiquid wealth (Z_t), and the value of the Markov process (ζ_t) that controls the AR(1) component of the income process. We denote the set of state variables by Λ_t. The non-redundant choice variables are I_t^X and I_t^Z. Consumption is a residual.

The household has constant relative risk aversion denoted by ρ. For $t \in \{20, 21, ..., 90\}$,[25] self t has instantaneous payoff function

$$u(C_t, Z_t, n_t) = n_t \cdot \frac{\left(\frac{C_t + \gamma Z_t}{n_t} \right)^{1-\rho} - 1}{1 - \rho}.$$

[25]By solving our model with annual periods, we are implicitly assuming that the additional β discount factor kicks in after one year for present-biased households (Laibson and Maxted, 2023). While one-year period lengths are likely longer than is psychologically realistic, we solve our model at an annual frequency for computational reasons. However, we conjecture that the choice of period length will affect our estimates of β to some extent, and in particular that credit card borrowing will be easier to generate for β values that are closer to 1 as the period length decreases (Maxted, 2022).

The household accumulates utility through consumption until death, at which point the household earns a bequest payoff of $B(t, X, Z)$.[26]

The quasi-hyperbolic discount function $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\}$ corresponds to a short-run discount rate of $-\ln(\beta\delta)$ and a long-run discount rate of $-\ln(\delta)$. When $\beta = 1$ the household discounts exponentially, which implies that household decisions are dynamically consistent. When $\beta < 1$, time inconsistency generates disagreement among selves.

Our benchmark in this paper follows Strotz (1956), Akerlof (1991), and O'Donoghue and Rabin (1999a,b) in adopting the assumption of "naivete." Under naivete, self t erroneously believes that all future selves will make choices that are aligned with self t's preferences. We assume naivete for computational simplicity, and note that our setting is not a good domain for identifying sophistication versus naivete. Indeed, Maxted (2022) proves an observational equivalence between naifs and sophisticates in a two-asset model similar to the one studied here. For completeness, in Section 7.2 we extend our analysis to "sophistication," where the current self is aware of future selves' present bias. Results are qualitatively similar.

Taking the (erroneously anticipated) strategies of other selves as given, self t picks a strategy at time t. This strategy is a map from the Markov state $\Lambda_t = \{t, X + Y, Z, \zeta\}$ to the choice variables $\{I^X, I^Z\}$. An equilibrium is a fixed point in the strategy space, such that all strategies are optimal given the perceived strategies of future selves.

For the naivete benchmark case, let $V^E_{t,t+1}(\Lambda_{t+1})$ represent self t's erroneous expectation of self $t+1$'s value function. Self t's objective function is

$$u(C_t, Z_t, n_t) + \beta\delta E_t V^E_{t,t+1}(\Lambda_{t+1}). \tag{3}$$

[26]We assume that bequeathed wealth is consumed by heirs as an annuity. Let $A(t, X, Z) = \max\{0, (R - 1)(X + (1 - \kappa_t)Z)\}$ denote the flow annuity payment from bequeathed wealth. This assumes that liquidating illiquid wealth entails the same transaction cost κ_t faced by a living household of age t. Let $\bar{n}$ and $\bar{y}$ denote average effective household size and average labor income over the lifecycle, respectively. We set bequest payoff $B(t, X, Z) = \frac{\alpha}{1-\delta}\left[u\left(\bar{y} + A(t, X, Z), 0, \bar{n}\right) - u(\bar{y}, 0, \bar{n})\right]$, where α is the weight placed on the bequest motive. Intuitively, this bequest motive captures the value of a household of size $\bar{n}$ increasing consumption from $\bar{y}$ to $\bar{y} + A(t, X, Z)$. Our baseline calibration sets $\alpha = 0.5$. See Appendix Table E4 for robustness.

Self t chooses $\left\{I^X, I^Z\right\}$ in state Λ_t to maximize this expression. Self t thinks that self $t+1$ will choose $\left\{I_{t+1}^{X,E}, I_{t+1}^{Z,E}\right\}$ to maximize an analogous expression, but with $\beta = 1$:

$$u(C_{t+1}, Z_{t+1}, n_{t+1}) + \delta E_{t+1} V_{t+1,t+2}^{E} \left(\Lambda_{t+2}\left(I_{t+1}^{X,E}, I_{t+1}^{Z,E}\right)\right). \tag{4}$$

For notational simplicity we denote $\Lambda_{t+2}\left(I_{t+1}^{X,E}, I_{t+1}^{Z,E}\right)$ as Λ_{t+2}^{E}. The sequence of continuation-value functions is defined recursively, where E superscripts always reflect naive expectations:

$$V_{t-1,t}^{E}(\Lambda_t) = (1 - \mathbb{1}_t^{death})[u(C_t^E, Z_t, n_t) + \delta E_t V_{t,t+1}^{E}(\Lambda_{t+1}^{E})] + \mathbb{1}_t^{death} B(\Lambda_t), \tag{5}$$

where $\mathbb{1}_t^{death}$ indicates that the household dies between period $t-1$ and t. We solve for equilibrium strategies using numerical backwards induction.

We generate $J_s = 10000$ independent streams of income realizations for J_s households, and we seed households with median age 20-24 wealth as calibrated from the SCF. Then we simulate lifecycle choices for these households, assuming they make equilibrium decisions conditional on their state variables. From the simulated profiles of C, X, Z, and Y, we calculate the moments used in the MSM estimation procedure. Note that the simulated profiles, and hence the summary moments, depend on the parameters of the model. Since the model cannot be solved analytically, its quantitative predictions are derived from the simulated lifecycle profiles.[27]

4 Two-Stage Method of Simulated Moments

We estimate the parameters of the model's discount function in the second stage of a Method of Simulated Moments procedure, closely following the methodology of Gourinchas and Parker (2002). MSM allows us to evaluate the predictions of our model and to formally

[27] See Maxted (2022) for a theoretical analysis of the effects of present bias on consumption-saving decisions.

Table 2: First-Stage Estimation Results

Demographics			Liquid Assets		
Number of children			*Credit limit* λ_t		
$k_t = \phi_0 exp(\phi_1 age - \phi_2 \frac{age^2}{100}) + \epsilon$			$\lambda_t = \phi_0 + \phi_1 age + \phi_2 \frac{age^2}{100} + \epsilon$		
ϕ_0	ϕ_1	ϕ_2	ϕ_0	ϕ_1	ϕ_2
0.003	0.358	0.508	0.167	-0.002	0.014
(4.21E-06)	(6.61E-05)	(8.69E-05)	(6.61E-02)	(3.18E-03)	(3.22E-03)
Number of dependent adults			*Real return on positive liquid assets R*		
$a_t = \phi_0 exp(\phi_1 age - \phi_2 \frac{age^2}{100}) + \epsilon$			1.0203		
			-		
ϕ_0	ϕ_1	ϕ_2			
4.59E-06	0.452	0.438	*Real credit card interest rate* R^{cc}		
(1.41E-08)	(1.24E-04)	(1.23E-04)	1.1059		
			-		

Illiquid Assets									
Consumption flow as a fraction of assets γ									
0.05									
-									
Real Income from Transfers and Wages									
Income process									
$y_t = ln(Y_t) = \phi_0 + \phi_1 age + \phi_2 \frac{age^2}{100} + \phi_3 \frac{age^3}{10000} + \phi_4 Nheads + \phi_5 Nchildren + \phi_6 Ndep.adults + \xi_t$									
$\xi_t = \eta_t + \nu_t = \psi \eta_{t-1} + \epsilon_t + \nu_t$									
ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ψ	σ_ϵ^2	σ_ν^2
7.563	0.135	-0.222	0.106	0.319	0.013	0.237	0.840	0.057	0.045
(0.256)	(0.017)	(0.037)	(0.024)	(0.023)	(0.008)	(0.012)	(0.047)	(0.005)	(0.004)

Source: Authors' estimation, following Laibson et al. (2003), based on data from the PSID, SCF, IPUMS-USA, FRB, and American Bankruptcy Institute. Note: Estimates pertain to households with heads who have high school diplomas but not college degrees. Standard errors are in parentheses. The constant of the deterministic component of income includes a birth-year cohort effect and a business cycle effect proxied by the unemployment rate. The dynamics of income estimation includes a household fixed effect. See Appendix B for estimation details. All return parameters are assumed to be exactly known in the context of the first stage. This table only reports standard errors, but the full covariance matrix is used in the second stage of the MSM procedure.

test the nested null hypothesis of exponential discounting, $\beta = 1$.[28] The current section describes our procedure. Appendix C presents derivations and additional technical details.

Our MSM procedure has two stages. In the first stage, nuisance parameters, χ, are estimated using standard techniques (see Table 2). We estimate these N_χ parameters and their associated variances, Ω_χ. Appendix B contains details.[29]

Given χ and Ω_χ, the second stage uses additional data and more of the model's structure to estimate N_θ additional parameters θ. The second stage, taking the first-stage parameters fixed at $\hat{\chi}$, chooses θ to minimize the distance between the empirical and the simulated moments. Specifically, we use the data from Section 2 on wealth accumulation and credit card borrowing over the lifecycle to estimate $\theta = (\beta, \delta, \rho)$ in the second stage. MSM, as we implement it, differs from a calibration exercise followed by a one-stage estimation in that it propagates uncertainty in the first-stage parameters into the standard errors of the second-stage parameter estimates. That is, the variance matrix of $\hat{\theta}$, denoted Ω_θ, depends on Ω_χ.[30] For parameters of the model that are not included in χ – like R, R^{CC}, γ, the parameters governing κ_t, and the bequest function – we perform additional robustness checks in Section 6.3.

Denote the empirical vector of N_m second stage aggregate moments by $\bar{m}_{J_m}$. Let J_m be the numbers of empirical observations used to calculate the elements of $\bar{m}_{J_m}$. Denote the theoretical population analogue to $\bar{m}_{J_m}$ by $m(\theta, \chi)$ and let $m_{J_s}(\theta, \chi)$ be the simulation approximation to $m(\theta, \chi)$. Let $g(\theta, \chi) \equiv [m(\theta, \chi) - \bar{m}_{J_m}]$ and $g_{J_s}(\theta, \chi) \equiv [m_{J_s}(\theta, \chi) - \bar{m}_{J_m}]$.

[28]See McFadden (1989), Pakes and Pollard (1989), and Duffie and Singleton (1993) for early formulations of MSM.

[29]Included in χ are income level coefficients, income variability coefficients, effective household size coefficients, and credit limit coefficients.

[30]Our derivation of Ω_θ assumes that the first-stage moments and the second-stage moments have uncorrelated measurement error. Most of the data that we use to identify θ and χ come from separate datasets. The only exception is the credit limit.

The moment conditions imply that in expectation

$$E[g(\theta_0, \chi_0)] = E[m(\theta_0, \chi_0) - \bar{m}_{J_m}] = 0,$$

where (θ_0, χ_0) is the true parameter vector. Define derivatives of the moment functions with respect to the parameters by $G_\theta \equiv \frac{\partial g(\theta_0, \chi_0)}{\partial \theta}$ and $G_\chi \equiv \frac{\partial g(\theta_0, \chi_0)}{\partial \chi}$. Let $\Omega_g \equiv E\left[g(\theta_0, \chi_0)\, g(\theta_0, \chi_0)'\right]$ be the variance of the second-stage moment estimates $\bar{m}_{J_m}$, which is estimated directly from sample data using bootstrapping.

Let W be a positive definite $N_m \times N_m$ weighting matrix. Define

$$q(\theta, \chi) \equiv g_{J_s}(\theta, \chi)' \cdot W \cdot g_{J_s}(\theta, \chi) \tag{6}$$

as a scalar-valued loss function, equal to the weighted sum of squared deviations of simulated moments from their corresponding empirical values. Our procedure is to fix χ at the value of its first-stage estimator, minimize the loss function $q(\theta, \hat{\chi})$ with respect to θ, and define the estimator as

$$\hat{\theta} = \arg\min_{\theta} \; q(\theta, \hat{\chi}). \tag{7}$$

As shown in Appendix C,

$$\Omega_\theta = Var\left(\hat{\theta}\right) = (G_\theta' W G_\theta)^{-1} G_\theta' W \left[\Omega_g + \Omega_g^s + G_\chi \Omega_\chi G_\chi'\right] W G_\theta (G_\theta' W G_\theta)^{-1}, \tag{8}$$

where $\Omega_g^s = \frac{J_m}{J_s}\Omega_g$ is the simulation correction. The first-stage correction is given by $G_\chi \Omega_\chi G_\chi'$. This correction increases with the uncertainty in our estimates of the first-stage parameters (Ω_χ) as well as the sensitivity of the second-stage moments to changes in the first-stage parameters (G_χ).

Equation (8) is used to calculate standard errors for our estimates of θ. All derivatives are replaced with numerical analogues, which we calculate using the model and simulation procedure. We estimate Ω_g and Ω_χ from sample data.

We use weighting matrix $W = diag\left(\hat{\Omega}_g\right)^{-1}$ for our baseline estimates. Many authors have found that optimally-weighted GMM procedures lead to biased estimates in small samples (e.g., Altonji and Segal, 1996; West et al., 2009). An important advantage of diagonal weighting matrices is that the contribution of each moment to $q(\theta, \hat{\chi})$ can be easily computed. We use this property in Section 6 to present a novel exploration of our model's identification, which we refer to as a *boundary analysis*. In robustness checks we find that our qualitative conclusions are not affected by the choice of weighting matrix.

5 Estimation Results

In this section we discuss the paper's main findings. We focus on estimates for the discount factors β and δ and the coefficient of relative risk aversion ρ, including the special case in which we impose $\beta = 1$. We also assess whether these models accurately predict key empirical regularities in the lifecycle literature summarized by the second-stage moments.

5.1 Benchmark Case

We report our benchmark estimates in Table 3. In the unconstrained case (Column 1), the MSM procedure yields an estimate of $\hat{\beta} = 0.530$ with a standard error (s.e.(i) in the table) of 0.114. For this specification, $\hat{\beta}$ lies significantly below 1; the t-stat for the $\beta = 1$ hypothesis test is $t = 4.1$. The MSM procedure yields an estimate of $\hat{\delta} = 0.989$, with a standard error of 0.005, and $\hat{\rho} = 1.936$, with a standard error of 0.435. The estimated values of β and δ imply a short-run discount rate of $-\ln(0.530 \cdot 0.989) = 64.5\%$ and a long-run discount rate of $-\ln(0.989) = 1.1\%$.

At the estimated parameter values, the present-biased model generates the moment pre-

Table 3: Benchmark Estimates

	(1) Present Biased	(2) Exponential	(3) Data
Parameter estimates			
$\hat{\beta}$	0.5305	1	-
s.e. (i)	(0.1143)	-	-
s.e. (ii)	(0.1128)	-	-
s.e. (iii)	(0.0536)	-	-
s.e. (iv)	(0.0503)	-	-
$\hat{\delta}$	0.9891	0.9601	-
s.e. (i)	(0.0051)	(0.0053)	-
s.e. (ii)	(0.0051)	(0.0053)	-
s.e. (iii)	(0.0022)	(0.0022)	-
s.e. (iv)	(0.0020)	(0.0020)	-
$\hat{\rho}$	1.9355	1.4663	-
s.e. (i)	(0.4350)	(0.2256)	-
s.e. (ii)	(0.4280)	(0.2228)	-
s.e. (iii)	(0.2188)	(0.1142)	-
s.e. (iv)	(0.2045)	(0.1085)	-
Second-stage moments			
% Visa 21-30	0.605	0.309	0.640
% Visa 31-40	0.585	0.287	0.629
% Visa 41-50	0.523	0.299	0.588
% Visa 51-60	0.475	0.257	0.503
mean Visa 21-30	0.103	0.044	0.111
mean Visa 31-40	0.117	0.051	0.096
mean Visa 41-50	0.124	0.059	0.110
mean Visa 51-60	0.116	0.034	0.104
wealth 21-30 | *debt*	0.913	0.880	1.222
wealth 31-40 | *debt*	1.412	0.999	1.868
wealth 41-50 | *debt*	2.640	1.941	3.377
wealth 51-60 | *debt*	4.723	3.811	4.650
wealth 21-30 | *no debt*	2.324	2.017	1.659
wealth 31-40 | *no debt*	3.248	2.850	2.800
wealth 41-50 | *no debt*	4.633	3.967	4.613
wealth 51-60 | *no debt*	7.475	5.358	8.071
Goodness-of-fit			
$q(\hat{\theta}, \hat{\chi})$	77.15	759.57	-
$\xi(\hat{\theta}, \hat{\chi})$	96.77	300.78	-
p-value	0	0	-

This table reports estimates of the discount function under our benchmark assumptions. The top half of the table presents parameter estimates and standard errors. The bottom half of the table reports the moments used to identify the second-stage parameters. Four standard errors are shown: (i) includes both the first-stage correction and the simulation correction, (ii) includes just the first-stage correction, (iii) includes just the simulation correction, and (iv) includes neither correction.

dictions reported in Column 1 of the lower panel of Table 3. Our estimated model has three free parameters, which are estimated with sixteen debt and wealth moments. We can compare the model's simulated moments with the sample moments, which are reproduced in Column 3. Qualitatively, the present-biased model successfully matches the lifecycle patterns of both credit card borrowing and wealth accumulation. Quantitatively, the model underpredicts the fraction borrowing in every decadal age bin. The model is quantitatively more successful in matching average credit card borrowing. The model also performs reasonably well in matching the age-based pattern of wealth formation – both for households with and without credit card debt – though misses to some extent early in the lifecycle.

We also estimate δ and ρ, imposing the restriction that $\beta = 1$. This exponential discounting case yields the results in Column 2. We find $\hat{\delta} = 0.960$, implying a discount rate of 4.1%, with a standard error of 0.005. We estimate $\hat{\rho} = 1.466$, with a standard error of 0.226.

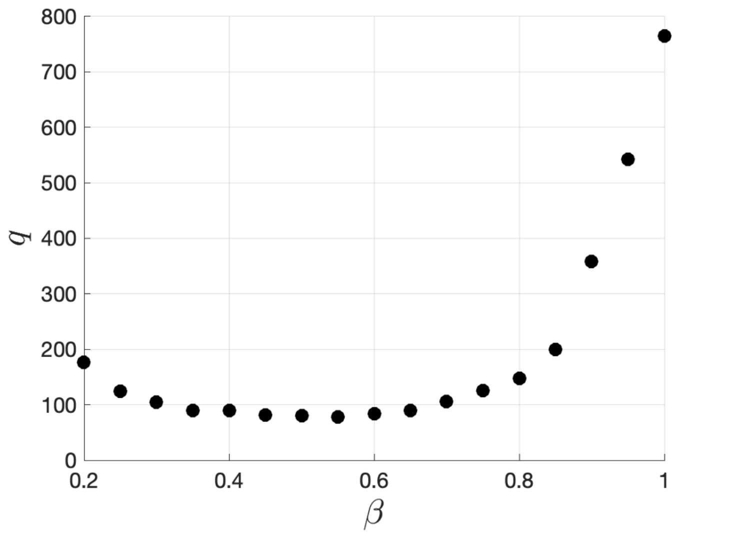
The exponential model matches the empirical facts about wealth accumulation over the lifecycle reasonably well, though it underpredicts wealth accumulation to some extent. Moreover, with such a low discount rate the model cannot account for observed credit card borrowing data. Instead, it predicts $\%Visa$ and $meanVisa$ to be relatively low at all ages. Intuitively, these credit card moments – which require a high discount rate – lose in the tug of war with the wealth moments – which require a low discount rate.[31] The best fit available under an exponential model predicts that most households have much less credit card debt than we observe in the data.

Figure 1 below provides visual evidence of the exponential model's inability to match the full set of empirical moments. Figure 1 fixes the value of $\beta \in \{0.2, 0.25, 0.3, \ldots, 0.95, 1\}$ and re-estimates the benchmark model for $\{\delta, \rho\}$ conditional on β. The vertical axis reports the q-value of best fit for each estimate. The model's ability to match the full set of credit card

[31] As we discuss in Section 6.3, if the credit card interest rate is low enough, or the return to illiquid wealth (i.e., γ) is high enough, the exponential model can more successfully match the facts simultaneously.

and wealth moments degrades as β approaches 1.

Figure 1: q on β (allowing δ and ρ to vary)



This figure illustrates the sensitivity of the model fit to restrictions on the short-run discount factor β. The vertical axis lists the MSM objective function q. Each point comes from a separate estimate of δ and ρ, conditional on the indicated β.

The standard errors reported as "s.e.(i)" in Table 3 and discussed above incorporate corrections for the first-stage estimation and for the simulation error. For comparison, we also report standard errors without these corrections: s.e.(ii) only includes the first-stage correction, s.e.(iii) only includes the simulation correction, and s.e.(iv) includes neither. The first-stage correction has a sizable effect on the standard errors. For example, if the first-stage parameters were known with certainty (comparing s.e.(i) and s.e.(iii)) then the standard error on β would roughly halve — falling from 0.114 to 0.054.

Finally, our two-stage MSM procedure enables us to provide formal over-identification tests that combine all of the simulated moments. Despite the present-biased model's better fit of the empirical moments, both models are rejected by over-identification tests. This is not surprising, given that we have 3 free parameters and 16 precisely estimated moments. For the present-biased model, the (inverse) goodness-of-fit measure is $\xi\left(\hat{\theta},\, \hat{\chi}\right) = 97$. For the exponential discounting model, the (inverse) goodness-of-fit measure is $\xi\left(\hat{\theta},\, \hat{\chi}\right) = 301$.

Under the null hypothesis that the model is correct, ξ is distributed chi-squared with degrees of freedom equal to the number of moments (16) minus the number of parameters (3 for the model with present bias and 2 for the exponential model). For reference, the 99% critical values of the chi-squared distribution with 13 and 14 degrees of freedom, respectively, are 27.7 and 29.1.

5.2 Benchmark Case: Model Properties

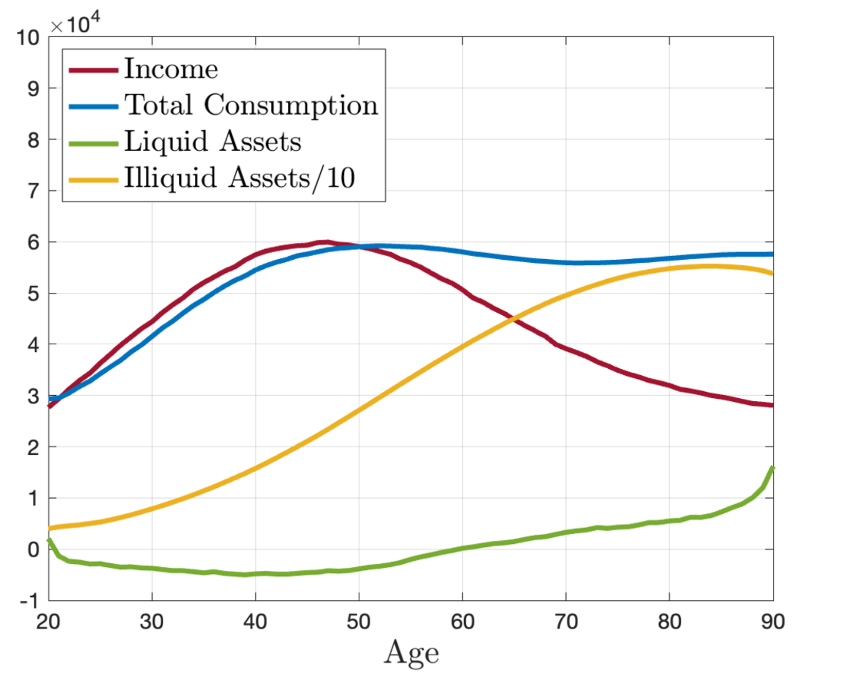
For our benchmark estimate with (naive) present bias, Figure 2 plots the average lifecycle profile of households in the model.[32] Average income peaks at age 47 before declining as households transition probabilistically into retirement. Even though income is declining, total consumption at the household level (which includes the consumption flow from the illiquid asset) remains approximately constant from age 45 until death. This is because the typical household accumulates illiquid wealth over the lifecycle. In particular, Appendix Figure 5 shows that households deposit into the illiquid asset – which pays a small premium over the liquid asset – during spells of higher income.[33] While households may naively anticipate not needing that liquidity in the medium run, the liquid-assets curve in Figure 2 illustrates that households' subsequent (over)consumption instead often leads them into credit card debt. Thus, the lifecycle profile in Figure 2 captures the underlying tension between acting patiently and acting impatiently that is reflected in our empirical moments: households accumulate large stocks of illiquid wealth, but also frequently carry credit card debt.

Our benchmark estimate also makes predictions about a household's marginal propensity to consume (MPC) over the lifecycle. In particular, we calculate the average one-year MPC

[32]For the $\beta = 1$ case, see Appendix Figure 4.

[33]Note that our model predicts too strong a relationship between households' income state and their propensity to invest in the illiquid asset. For example, our model does not include various sorts of "forced savings" channels, such as monthly mortgage payments, that do not vary at high frequency (in part due to contractual features that may exist as an endogenous response to self-control problems).

Figure 2: Average Lifecycle Profile for Present-Biased Estimate



This figure plots the average lifecycle profile of income, total consumption, liquid assets, and illiquid assets (divided by ten for scaling) for the benchmark estimate ($\hat{\beta} = 0.530$).

out of a \$1000 windfall.[34] For age groups [21-30, 31-40, 41-50, 51-60] the average annual MPC is [0.31, 0.28, 0.19, 0.13], respectively. Using the method of Laibson et al. (2021b), we can also convert the model's MPCs into marginal propensities for expenditure (MPXs), which include total spending on both nondurables and consumer durables. For age groups [21-30, 31-40, 41-50, 51-60] the average annual MPX is [0.45, 0.41, 0.27, 0.18], respectively.[35]

This pattern of declining MPCs and MPXs fits with the lifecycle predictions of the model. As households age they accumulate illiquid wealth and face a lower transaction cost

[34] We use a non-uniform grid when solving the model numerically, and for some high-liquidity households the grid increment is greater than \$1,000. In these cases, we impute the \$1,000 MPC.

[35] Laibson et al. (2021b) show that the one-period MPX can be approximated by:

$$MPX = \left(1 - s + \frac{s}{r + \mathcal{V}}\right) \times MPC,$$

where s is the durable share of consumption, $\mathcal{V}$ is the (annual) depreciation rate, and r is the interest rate. We set $r = 2.03\%$ (see Table 2), and follow the calibration of Laibson et al. (2021b) in setting $s = 0.125$ and $\mathcal{V} = 0.199$.

to access this wealth, both of which decrease the share of hand-to-mouth households over the lifecycle.[36] The model's predicted MPX age dynamics are also consistent with the empirical evidence in Fagereng et al. (2021), who document a tendency for the MPX out of lottery winnings to decline with age.

6 Identification

6.1 Boundary Analysis

A drawback to structural modeling is that the forces driving parameter identification are often opaque. In this section we present a novel strategy – which we call a *Boundary Analysis* – in order to address this identification challenge. The goal of our boundary analysis is to restrict the three-dimensional $\theta = (\beta, \delta, \rho)$ parameter space to areas where the simulated moments are close to their empirical counterparts. These boundaries help us to detect the underlying trade-offs that the MSM procedure makes as it chooses $\hat{\theta}$ optimally to fit the empirical moments.[37]

To conduct this analysis, we create a discrete three-dimensional grid of $\theta = (\beta, \delta, \rho)$ values and solve our model at each grid point θ_i. Our choice of a diagonal weighting matrix means that it is easy to determine the contribution of each individual moment (or set of moments) to $q(\theta_i, \hat{\chi})$.[38] We group our 16 moments into two categories – credit card borrowing moments and wealth moments – and determine each category's contribution to $q(\theta_i, \hat{\chi})$. Let $q_{cc}(\theta_i, \hat{\chi})$ denote the contribution of the credit card moments and let $q_{wealth}(\theta_i, \hat{\chi})$ denote the

[36] Related to this prediction, Leth-Petersen (2010) and Sodini et al. (2023) show that young households are more responsive to credit supply increases.

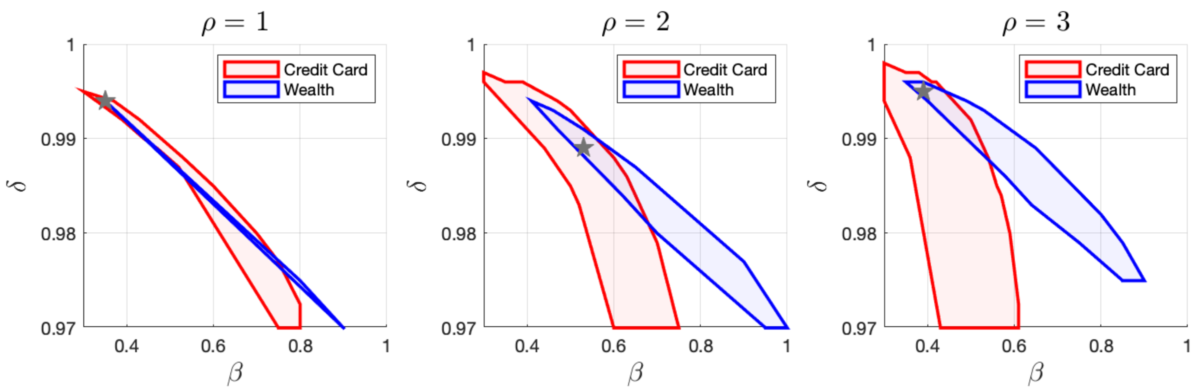
[37] The discussion of identification in this section will focus on how $\hat{\theta}$ is chosen to match the empirical moments of Section 2. Alternatively, a reader could ask how changes to the empirical moments affect the estimate of $\hat{\theta}$. To address this question we report the Andrews et al. (2017) sensitivity measure of $\hat{\theta}$ in Appendix Table D1.

[38] Objective function $q(\theta, \hat{\chi}) = g_{J_s}(\theta, \hat{\chi})' \cdot W \cdot g_{J_s}(\theta, \hat{\chi})$, so W being diagonal implies that $q(\theta, \hat{\chi})$ is just a weighted sum of squared errors.

contribution of the wealth moments, such that $q(\theta_i, \hat{\chi}) = q_{cc}(\theta_i, \hat{\chi}) + q_{wealth}(\theta_i, \hat{\chi})$.

The boundary analysis is presented in Figure 3. In order to visualize the three-dimensional grid of θ values we show three two-dimensional plots in (β, δ)-space for $\rho \in \{1, 2, 3\}$. Each plot has two boundaries. The red boundary encases the set of grid points at which $q_{cc} < 77$, and the blue boundary encases the set of grid points at which $q_{wealth} < 77$. The threshold of 77 is chosen to align with the q of our baseline estimate in Table 3. The stars mark the point of best fit conditional on ρ.

Figure 3: Boundary Analysis



This figure shows the boundary analysis. The red locus marks the set of points in (β, δ)-space for which $q'_{cc} \cdot W_{cc} \cdot q_{cc} < 77$. The blue locus marks the set of points for which $q'_{wealth} \cdot W_{wealth} \cdot q_{wealth} < 77$. The star marks the point of best fit conditional on ρ.

We now use this boundary analysis to aid our discussion of the identification of $\hat{\theta}$. Starting with $\hat{\rho}$, Figure 3 illustrates two effects that occur as ρ increases. The first effect is that the two boundaries pull apart — the red credit card boundary shifts left and down toward lower values of β and δ while the blue wealth boundary shifts right and up toward higher values of β and δ. This first effect calls for the estimation of a low ρ in order to keep the two boundaries close together. On the other hand, the second effect of increasing ρ is that the area enclosed by each boundary increases. Put differently, the gradient of q_{cc} and q_{wealth} with respect to β and δ is decreasing in ρ. This second effect calls for the estimation of a high ρ. Balancing these two effects yields $\hat{\rho} \approx 1.9$ as is estimated in Table 3.

To understand the first effect of boundaries pulling apart, a higher value of ρ increases the household's precautionary savings motive. This discourages credit card borrowing in order to preserve that adjustment margin for a series of negative income shocks. As ρ increases the model can only match the empirical credit card borrowing moments with lower discount factors, hence the shift left and down of the red boundary. On the other hand, a higher value of ρ also discourages wealth accumulation beyond what is needed for self-insurance purposes. Wealth accumulation involves trading off current consumption for future consumption, and a higher ρ makes this trade-off less appealing. Thus, as ρ increases the blue boundary shifts right and up because higher discount factors are needed to continue fitting the empirical wealth moments.

For the second effect of decreasing gradients, recall that ρ is the inverse of the EIS, such that increasing ρ decreases the sensitivity of consumption growth to β and δ.[39] Thus, conditional on being at a grid point θ_i that is included in either the credit card or wealth boundary, larger changes to β and δ will be required for q_{cc} or q_{wealth} to change by enough to exit the boundary.

Now that we've identified ρ, we discuss the identification of $\hat{\beta}$ and $\hat{\delta}$ conditional on $\rho \approx 1.9$. Here we focus on the $\rho = 2$ subplot. The shape of both the red and blue areas implies that β and δ are partial substitutes: when δ is high then low values of β best match the empirical moments, and vice-versa. Nonetheless, β and δ are separately identified because they are not perfect substitutes. The divergence of the red and blue boundaries as β approaches 1 shows that the effect of δ relative to β is larger for wealth accumulation than for credit card borrowing.[40] Matching the wealth moments relies on substantial *illiquid* asset accumulation, while matching the credit card moments relies on minimal *liquid* asset accumulation. Illiquid assets have a longer effective horizon than liquid assets, hence giving

[39] The EIS is not exactly equal to the inverse of ρ for two reasons. First, on the equilibrium path, households sometimes face binding liquidity constraints. Second, present bias can introduce a wedge between the EIS and the inverse of ρ (see e.g., Laibson, 1998).

[40] As β increases toward 1, δ needs to fall by only a little to continue fitting the wealth moments. Alternatively, δ needs to fall by a lot to continue fitting the credit card moments.

δ relatively more influence on them. As the $\rho = 2$ subplot shows, for β near 1 our model cannot fit both wealth and credit card moments, but by decreasing β (and increasing δ accordingly) our model is able to generate the simultaneous credit card borrowing and wealth accumulation required to match the patterns we observe in the data.[41]

6.2 Moment Sets

As is well known from GMM theory, the choice of moments can be crucial to the outcome of the analysis. We chose to focus on the 16 moments discussed above – each of $\%Visa$, $meanVisa$, $wealth|debt$, and $wealth|no\ debt$, in four decadal age bins – because of their economic importance and their transparency. Since our model is overidentified, if it is correctly specified then omitting some of these moments should not change our estimated parameter values.

We pursue an initial examination of restrictions to the moment set in Table 4. Column 1 of Table 4 repeats the benchmark results. Columns 2-5 drop the $\%Visa$, $meanVisa$, $wealth|debt$, and $wealth|no\ debt$ groups of moments, respectively. While our estimates vary to some extent across these cases, we consistently estimate $\hat{\beta} < 1$.

Column 6 takes the more aggressive step of dropping both the $\%Visa$ and $meanVisa$ moment blocks simultaneously. In this case, we estimate $\hat{\beta} = 0.761$ and $\hat{\delta} = 0.980$. The wealth accumulation moments are matched well, but the model fails to match the credit card moments. In Column 7 we report estimates when all eight *wealth* moments are dropped. This results in $\hat{\beta} = 0.483$ and $\hat{\delta} = 0.989$. Now, the credit card moments are matched well, but there is too little lifecycle wealth accumulation. Together with the boundary analysis above, Columns 6 and 7 help to illustrate that identification comes from the inclusion of both types of moments in order to generate a tension between short-run and long-run behavior.

[41] See also Maxted (2022) and Lee and Maxted (2023) for further theoretical analysis of how β versus δ affect credit card borrowing and illiquid wealth accumulation.

Table 4: Sources of Identification

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		-	$\%Visa$	$\%Visa$	$\%Visa$	-	$\%Visa$	
		$meanVisa$	-	$meanVisa$	$meanVisa$	-	$meanVisa$	
	Benchmark	$wealth\|debt$	$wealth\|debt$	-	$wealth\|debt$	$wealth\|debt$	-	Data
		$wealth\|no\ debt$	$wealth\|no\ debt$	$wealth\|no\ debt$	-	$wealth\|no\ debt$	-	
Parameter estimates								
$\hat{\beta}$	0.5305	0.6593	0.5309	0.2723	0.4088	0.7612	0.4827	-
s.e. (i)	(0.1143)	(0.0760)	(0.0965)	(0.0602)	(0.0928)	(0.0871)	(0.1638)	-
$\hat{\delta}$	0.9891	0.9844	0.9886	0.9975	0.9925	0.9798	0.9893	-
s.e. (i)	(0.0051)	(0.0039)	(0.0042)	(0.0016)	(0.0031)	(0.0053)	(0.0036)	-
$\hat{\rho}$	1.9355	1.9548	1.7870	3.9063	1.1567	1.8133	2.4276	-
s.e. (i)	(0.4350)	(0.3582)	(0.6178)	(0.1327)	(0.1090)	(0.4516)	(1.5034)	-
Second-stage moments								
% Visa 21-30	0.605	0.504	0.612	0.642	0.611	0.439	0.629	0.640
% Visa 31-40	0.585	0.498	0.595	0.606	0.587	0.435	0.609	0.629
% Visa 41-50	0.523	0.479	0.540	0.524	0.527	0.442	0.564	0.588
% Visa 51-60	0.475	0.436	0.497	0.437	0.481	0.414	0.497	0.503
mean Visa 21-30	0.103	0.082	0.106	0.113	0.109	0.068	0.109	0.111
mean Visa 31-40	0.117	0.094	0.120	0.121	0.124	0.079	0.124	0.096
mean Visa 41-50	0.124	0.107	0.130	0.121	0.133	0.095	0.135	0.110
mean Visa 51-60	0.116	0.090	0.125	0.103	0.148	0.077	0.117	0.104
wealth 21-30 | debt	0.913	0.951	0.922	0.648	1.087	0.994	0.774	1.222
wealth 31-40 | debt	1.412	1.559	1.412	1.017	1.619	1.634	1.140	1.868
wealth 41-50 | debt	2.640	2.892	2.588	2.207	2.835	3.022	2.170	3.377
wealth 51-60 | debt	4.723	5.213	4.553	4.662	4.776	5.301	4.003	4.650
wealth 21-30 | no debt	2.324	2.236	2.348	2.069	2.515	2.210	2.188	1.659
wealth 31-40 | no debt	3.248	3.186	3.260	2.786	3.590	3.173	2.891	2.800
wealth 41-50 | no debt	4.633	4.689	4.628	4.288	4.937	4.649	4.128	4.613
wealth 51-60 | no debt	7.475	7.246	7.377	7.638	7.672	7.014	6.528	8.071
Goodness-of-fit								
$q(\hat{\theta}, \hat{\chi})$	77.15	53.22	72.85	25.61	29.04	43.18	8.17	-

This table reports estimates of the discount function under alternate moment sets. Columns (2) through (7) vary the set of empirical moments used for estimation by dropping the shaded moments, maintaining all other benchmark assumptions. In Columns (2), (3), (4), and (5), we remove each of the four moment sets individually. In Column (6), we estimate using two conditional wealth moments, $wealth|debt$ and $wealth|no\ debt$. In Column (7), we estimate using two credit card borrowing moments, $\%Visa$ and $meanVisa$.

6.3 Robustness

In Appendix E we explore robustness to a variety of first-stage parameters and structural modeling assumptions. In summary, estimates of β reliably fall between roughly 0.3 and 0.8. Table E1 fixes the coefficient of relative risk aversion ρ to a variety of common values and re-estimates β and δ conditional on the specified ρ. Table E2 examines robustness to alternate assumptions on return parameters R, R^{CC}, and γ. Table E3 explores robustness to the model's income process, and Table E4 studies robustness to other structural modeling assumptions.[42] Estimates using alternate weighting matrices are presented in Table E5.

[42]This includes the liquidity of the Z account, assumptions about household returns to scale, and the strength of the bequest motive.

Appendix E also provides details on our choices for robustness checks.

The one robustness analysis that we discuss in the main text is the sensitivity of household savings behavior to joint assumptions on R^{CC} and γ. We base this analysis on the seminal model of Kaplan and Violante (2014), which also features a liquid asset and an illiquid asset, but is able to generate sizable borrowing with $\beta = 1$. Their calibration sets the real return on liquid wealth to -1.48%, the real return on illiquid wealth to 2.29% plus a 4% consumption flow (for a total return of 6.29%), and the real cost of borrowing to 6%. In our model, this corresponds roughly to $R = 0.9852$, $\gamma = 6.29\%$, and $R^{CC} = 1.06$. In Table E6 we estimate our model under this alternate calibration, finding $\hat{\beta} = 1.04$ and $\hat{\delta} = 0.95$. These estimates are very similar to the calibration of Kaplan and Violante (2014), who set $\beta = 1$ and $\delta = 0.94$.[43] This result highlights that short-term borrowing can be rationalized if there is a small (or negative) wedge between the cost of borrowing and the return on illiquid wealth (e.g., $r^{CC} = 6\%$ and $\gamma = 6.29\%$). However, $\beta < 1$ allows the model to generate significant credit card borrowing even when there is a large wedge between the cost of borrowing and the return on illiquid wealth (e.g., $r^{CC} = 11\%$ and $\gamma = 5\%$). For additional intuition, Appendix Figure 6 provides a boundary analysis with respect to the model's interest rates. This analysis further illustrates that the borrowing moments can be fit at higher values of β as the wedge between the cost of borrowing and the return on illiquid wealth declines.

We emphasize one interesting caveat to this discussion, which is that household beliefs may also be important. In particular, while the above analysis shows that empirically realistic levels of short-term borrowing can be generated without present bias if there is a small wedge between the cost of borrowing and the return on illiquid wealth, it may be sufficient for there to be a small wedge between households' *perceived* cost of borrowing and/or their *perceived* return on illiquid wealth. We explore a preliminary analysis of this by varying households'

[43] Kaplan and Violante (2014) set $\beta = 1$ and calibrate R^{CC} internally to match the credit card borrowing observed in the data. In this paper, we instead take R^{CC} from the data and allow β to adjust in order to match the credit card borrowing observed in the SCF.

perceived return on illiquid wealth (while holding the realized return at 5%). As Table E7 shows, the $\beta = 1$ model's fit of the data improves as households' perceived return on the illiquid asset approaches the credit card borrowing rate.

7 Extensions

This paper's findings suggest several directions for future work. This section discusses three such directions and provides initial analyses. Results are reported in Appendix F.

7.1 Heterogeneity

One important avenue to explore involves relaxing the assumption of homogeneous preferences.[44] Certainly there is substantial heterogeneity in the population. One might wonder whether a model with exponential consumers, but with heterogeneous δ's, could also resolve the empirical tensions discussed in this paper.

To explore this question, we fix $\beta = 1$ and $\rho \in \{0.5, 1, 2, 3\}$, and allow for preference heterogeneity over δ. In particular, let F_δ denote the CDF of the distribution over δ, which we assume follows a beta distribution.[45]

To estimate the parameters of F_δ we follow a process analogous to the MSM procedure used throughout the paper. Let F denote an arbitrary CDF of the joint distribution of $\theta = (\beta, \delta, \rho)$. Let

$$m_h(F, \chi) = \int m_{J_s}(\theta, \chi)\, dF(\theta),$$

[44] For lifecycle models exploring preference heterogeneity, see e.g., Gomes and Michaelides (2005), Vestman (2019), and Calvet et al. (2021).

[45] We choose a beta distribution to ensure that $\delta \in [0, 1]$.

and

$$\hat{F} = \arg\min_{F} \left(m_h\left(F, \hat{\chi}\right) - \bar{m}_{J_m}\right)' W \left(m_h\left(F, \hat{\chi}\right) - \bar{m}_{J_m}\right).$$

The main challenge for implementation is computation of the integral defining m_h, since each calculation of $m_{J_s}(\theta, \chi)$ is numerically costly.[46] We approximate the integral by calculating $m_{J_s}(\theta, \chi)$ over a dense grid of θ values and discretizing F over that grid.

Our estimates of the exponential heterogeneity cases are reported in Table F1. When $\rho = 2$ (the best-fitting case), the estimated beta distribution for δ has a mean of 0.87 with substantial dispersion. Allowing for heterogeneity in δ generates $q(\hat{F}, \hat{\chi}) = 364.27$. While this is a large improvement on the $\beta = 1$ single-point estimate (Column 2 of Table 3), the exponential model with heterogeneity still underperforms the single-point estimate that allows for $\beta < 1$. As Table F1 shows, the exponential heterogeneity model is simply unable to match the observed wealth levels for households carrying credit card debt. Though heterogeneity is obviously important, this suggests that some applications may achieve better performance at lower computational cost by incorporating present bias.

7.2 Sophistication

The benchmark model in this paper generalizes exponential discounting to allow for naive present bias. Under naivete the current self is unaware of future selves' present bias, and instead believes that future selves will discount exponentially. An alternative is to assume that the current self is (at least partially) aware of the present bias of future selves and hence perceives the self-control problem.

Harris and Laibson (2001) show that strategic interactions between the temporal selves of a sophisticated consumer induce pathological discontinuities in policy functions and kinks in

[46] Throughout this analysis, another limitation is the assumption that heterogeneity in (β, δ, ρ) is uncorrelated with unobserved heterogeneity in the first-stage parameters.

value functions.[47] Solutions to the (finite horizon) lifecycle problem are still computable by backward induction, so we report estimates of $\hat{\beta}$ and $\hat{\delta}$ under the sophistication assumption in Table F2.

One consequence of the pathologies induced by sophistication is that $q(\theta, \hat{\chi})$ may not be continuous and therefore three caveats apply to the results in Table F2. First, the results are numerically more fragile than under naivete. Our estimates in Table F2 are presented conditional on ρ in order to offset this fragility by reducing the dimensionality of the minimization problem. Second, the numerical instability arising from pathologies means that any low-q points that are found could be partially a result of spurious fit. Third, our calculation of standard errors assumes differentiability of the objective function (see Appendix C for details). Standard errors are grayed out in Table F2 to indicate that this assumption is not met under sophistication. With these caveats in mind, the results in Table F2 show that our identification of $\hat{\beta} < 1$ does not rely on the assumption of naivete.

7.3 Educational Attainment

In this paper we focus on one educational category: high school graduates. By updating our first- and second-stage moments we are also able to estimate the discount functions of other educational populations.[48] Table F3 reports estimates of θ for the population without a high school degree, and the population with a college degree. Table F3 also includes the targeted empirical moments for these other educational populations, which are calculated using the same methodology as our benchmark moments. Our point estimates for $\hat{\beta}$ are less than 1 in both cases, though with larger standard errors than our baseline estimate (e.g., for the population with a college degree, we estimate $\hat{\beta} = 0.79$ with a standard error of 0.22).

[47] See also Laibson and Maxted (2023) for a discussion.

[48] Certain structural assumptions of our model were made with the high school graduate demographic in mind, so some of our assumptions do not map cleanly into other educational groups. For example, it would be preferable to start a model of college graduates at age 23 rather than age 20.

8 Conclusion

This paper uses a structural lifecycle model to estimate household time preferences. In the data, U.S. households accumulate substantial illiquid wealth before retirement while simultaneously borrowing actively on credit cards. To explain these phenomena our MSM procedure estimates $\hat{\beta} = 0.53$, $\hat{\delta} = 0.99$, and $\hat{\rho} = 1.94$. The low long-run discount rate $(-\ln\delta = 1.1\%)$ accounts for observed levels of (illiquid) wealth accumulation. The high short-run discount rate $(-\ln\beta\delta = 64.5\%)$ explains the observed levels of credit card borrowing. The MSM procedure rejects the restriction to exponential discounting ($\beta = 1$) in the benchmark parametrization and almost all robustness checks. The model with $\beta = 1$ can replicate either the wealth data or the credit card borrowing data, but not both.

The evidence reported here suggests that present bias improves the ability for consumption-saving models to match household balance sheet data over the lifecycle. Quantitatively, our parameter estimates are sensitive to some calibrational choices and our economic environment is highly stylized. One path for future research is to enrich the realism of our modeling framework. Additionally, counterfactual policy analysis in a model similar to the one studied here may be a fruitful avenue for research.

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** INTERNET APPENDIX **

A Second-Stage Moments Appendix

We now discuss our procedure for constructing the $\%Visa$, $meanVisa$, $wealth|debt$, and $wealth|no\ debt$ moments. We use the 1995 through 2013 SCF waves and all quantities are deflated to 2010 dollars. Throughout, we impose the following sample restrictions: the head of house has a high school degree and no college degree, the head is between the ages of 21 and 90, the household possesses a credit card, the household earns an annual income of at least \$1000, and neither the head nor spouse is self-employed nor earns private business income. We control for household demographics, business cycle effects, and cohort effects to align the SCF population with our model's simulated population. We assign to households in our simulations the mean empirical demographic, business cycle, and cohort effects. All of our measures of credit card debt only include debt on which the household reports paying an interest rate above 5%, following Lee and Maxted (2023).

For each variable of interest x we first use weighted least squares, applying the SCF population weights, to estimate

$$x_i = FE_i + BCE_i + CE_i + AE_i + \xi_i. \tag{9}$$

FE_i is a family size effect that consists of three variables: the number of heads, the number of children under 18, and the number of dependent adults in household i. BCE_i is a business cycle effect proxied by the unemployment rate in the household's region of residence. In the 1995 and 1998 SCF waves we control for the unemployment rate in the household's Census Division. In all other years the nationwide rate is used because information on household location is not available in the public use dataset. CE_i is a cohort effect that consists of a full set of five-year cohort dummies, AE_i is an age effect that consists of a full set of age dummies, and ξ_i is an error term.[49]

Next, we define the "typical" household to be identical to the simulated household (i.e., with head and spouse, exogenous age-varying numbers of children and adult dependents, a fixed cohort effect, and an average unemployment effect).[50] To capture the "typical" household empirically we create a new variable, $\widehat{x}_i$, that captures what x_i would have been had household i been typical. For example, if household i is identical to the "typical" household except for having more children, we set $\widehat{x}_i = x_i + \phi(\overline{nkids_t} - nkids_i)$, where ϕ is the coefficient on number of children in regression (9) and $\overline{nkids_t}$ is the typical number of children in a household (as a function of the head's age). As detailed below, all second-stage moments are estimated using $\widehat{x}_i$. Standard error calculation follows the procedure outlined in the 2013 SCF Codebook, and adjusts for both imputation and sampling error.

To construct the $\%Visa$ moments we create a dummy variable $hasdebt_i$ equal to one for household i if the household has a positive outstanding credit card balance in the SCF. We apply the "typical" household adjustment to $hasdebt_i$ in order to generate $\widehat{hasdebt}_i$. To estimate the four $\%Visa$ moments, we separately calculate a weighted average of $\widehat{hasdebt}_i$ for each age bin 21-30, 31-40, 41-50, and 51-60.

Construction of $meanVisa$ is complicated by the fact that aggregate credit card data from the Fed indicates that SCF borrowing magnitudes are severely biased downward (Zinman, 2009; Beshears et al., 2019). We correct for this bias by scaling all credit card debt in the SCF by parameter $\alpha = 2$.[51] Let $debt_i = \alpha \cdot debt_i^{SCF}$, where $debt_i^{SCF}$ is the level of credit card debt reported in the SCF for household i. We calculate age-specific income means ($\overline{y}_t$) and

[49] Following Gourinchas and Parker (2002) we attribute time effects to fluctuations in unemployment, but this approach – like any approach to separate age, cohort, and time effects – requires problematic identification assumptions.

[50] Refer to Appendix B on the first-stage parameters for details. Age-varying numbers of children and adult dependents are estimated using IPUMS-USA data. The average unemployment rate is the one used in the calibration of the income process (PSID). To account for the rising usage of credit cards we adjust all households to the cohort born between 1980 and 1984.

[51] When computing the sampling error we assume that α has a standard deviation of 0.5.

define $debtinc_i$ as $debt_i/\overline{y}_t$.[52] We correct $debtinc_i$ to generate $\widehat{debtinc}_i$. The four $meanVisa$ moments are the weighted average of $\widehat{debtinc}_i$ for each of our decadal age buckets.

For $wealth|debt$ and $wealth|no\ debt$ we include in the numerator all real and financial wealth (e.g., home equity and CDs) as well as all claims on defined-contribution pension plans (e.g., 401(k)s). The measure does not include Social Security wealth and claims on defined-benefit pension plans, which appear in our estimated income process. After generating the wealth numerator (w_i) for each household i we define $wealthinc_i$ as $w_i/\overline{y}_t$. We correct $wealthinc_i$ to generate $\widehat{wealthinc}_i$. In order to estimate the four $wealth|debt$ moments and the four $wealth|no\ debt$ moments, we separately calculate a weighted average of $\widehat{wealthinc}_i$ for households with and without credit card debt for each of our decadal age buckets.

B First-Stage Parameters Appendix

This Appendix outlines our methodology for estimating the first-stage parameters used in our MSM procedure. This methodology is based on Laibson et al. (2003). We can separate the first-stage parameters into household demographics, the real income process, and the credit limit. Throughout, we impose the following sample restrictions: the head of house has a high school degree and no college degree, the head is between the ages of 20 and 90, the household earns an annual income of at least $1000, and neither the head nor spouse is self-employed nor earns private business income. Income is deflated to 2010 dollars.

B.1 Household Demographics

Household demographics are estimated using the decennial census in 1980, 1990, and 2000, and the American Community Survey from 2001 through 2014 (Ruggles et al., 2010). In addition to the restrictions above, we limit the sample to households with a head and spouse.

[52]When calculating the age-specific income means we group together ages 71-75, 76-80, and over 80 because we have fewer observations at those ages.

Children are defined as household members younger than 18. Dependent adults are defined as household members, other than the head and spouse, who are at least 18 years old.

On this sample we use weighted nonlinear least squares to estimate the following model:

$$x_i = \phi_0 \cdot \exp\left(\phi_1 \cdot age_i - \phi_2 \cdot \frac{age_i^2}{100}\right) + \varepsilon_i.$$

Dependent variable x_i equals either the number of children or the number of dependent adults in household i.

B.2 Income Process

The real income process is estimated using data from the PSID. We use years 1982 through 1990 because these years include complete data on income and federal taxes. We estimate the following regression using PSID family weights:

$$y_{it} = \text{cons.} + \text{polynomial}(age_{it}) + FE_{it} + BCE_{it} + CE_i + h_i + \xi_{it}.$$

Variable y_{it} is the log of after-tax real income from transfers and wages for household i in year t.[53] The cubic age polynomial is defined as $\text{polynomial}(age_{it}) = \phi_1 \cdot age_{it} + \phi_2 \cdot \frac{age_{it}^2}{100} + \phi_3 \cdot \frac{age_{it}^3}{10000}$, where age_{it} is the age of the head of household i in year t. Family size effect $FE_{it} = \phi_4 \cdot Nheads_{it} + \phi_5 \cdot Nchildren_{it} + \phi_6 \cdot Ndep.adults_{it}$, where $Nheads_{it}$ is the number of heads of house (head only or head and spouse), $Nchildren_{it}$ is the number of children, and $Ndep.adults_{it}$ is the number of dependent adults in household i in year t. Business cycle effect BCE_{it} is proxied by the unemployment rate in the household's state of residence. CE_i is a birth year cohort effect for the head of house that consists of a full set of five-year cohort dummies. h_i is a household-level fixed effect. Note that the constant ϕ_0 reported in Table 2

[53] Data on the income of household i in year t is collected by the PSID in survey year $t+1$, since PSID interview questions relate to the previous year's income.

includes the average business cycle and cohort effect. Standard errors are bootstrapped.

The first six terms of the income process listed in Table 2, corresponding to parameters ϕ_0 through ϕ_6, represent the deterministic component of household income in our simulation. In order to add a stochastic component to the simulated income process, we model error term ξ_{it} as the sum of an AR(1) process and a transitory shock:

$$\xi_{it} = \eta_{it} + \nu_{it} = (\psi \cdot \eta_{it-1} + \varepsilon_{it}) + \nu_{it}.$$

The parameters ψ, σ_ε^2, and σ_ν^2 reported in Table 2 are estimated using an equally weighted GMM procedure that minimizes the distance between the first seven theoretical and empirical autocovariances of $\Delta\xi$ (see Laibson et al. (2003) for details). Standard errors are bootstrapped. When solving the model numerically, we approximate the AR(1) process with three persistent income states.

B.3 Credit Limit

SCF data is used to estimate the age-dependent credit limit. We divide SCF-reported credit card limits by age-specific mean income $\bar{y}_t$ and then apply the typical household adjustment (as described in Appendix A). On this adjusted credit limit to income ratio we fit a quadratic age polynomial. Estimates are reported in Table 2. Standard errors are calculated following the procedure outlined in the 2013 SCF Codebook.

C MSM Procedure Appendix

Since $m(\theta, \hat{\chi})$ is difficult to evaluate analytically we replace it with an unbiased simulator, calculated by taking J_s draws of the initial distribution and then constructing the corresponding simulated expectations. Define $m_{J_s}(\theta, \hat{\chi})$ as the vector of simulated moments. We can find the vector $\hat{\theta}$ that minimizes $g'_{J_s}(\theta, \hat{\chi}) W g_{J_s}(\theta, \hat{\chi})$, where $g_{J_s}(\theta, \hat{\chi}) = m_{J_s}(\theta, \hat{\chi}) - \bar{m}_{J_m}$.

The first order condition for the second stage is given by

$$g'_{J_s\theta}\left(\hat{\theta}, \hat{\chi}\right) W g_{J_s}\left(\hat{\theta}, \hat{\chi}\right) = 0,$$

where $g_{J_s\theta}\left(\hat{\theta}, \hat{\chi}\right) = \partial g_{J_s}\left(\hat{\theta}, \hat{\chi}\right) / \partial\theta'$.

Following Gourinchas and Parker (2002) and Newey and McFadden (1994), an expansion of $g_{J_s}\left(\hat{\theta}, \hat{\chi}\right)$ around θ_0 to first order leads to

$$g'_{J_s\theta}\left(\hat{\theta}, \hat{\chi}\right) W \left[g_{J_s}(\theta_0, \hat{\chi}) + g_{J_s\theta}(\theta_0, \hat{\chi})\left(\hat{\theta} - \theta_0\right)\right] = 0.$$

Rearranging terms and defining $\hat{J}_m$ as the (scalar) rate of convergence of $\hat{\theta}$,

$$\sqrt{\hat{J}_m}\left(\hat{\theta} - \theta_0\right) = -\left[g'_{J_s\theta}\left(\hat{\theta}, \hat{\chi}\right) W g_{J_s\theta}(\theta_0, \hat{\chi})\right]^{-1} g'_{J_s\theta}\left(\hat{\theta}, \hat{\chi}\right) W \sqrt{\hat{J}_m} g_{J_s}(\theta_0, \hat{\chi}).$$

Let $\Pi \equiv \left[g'_{J_s\theta}\left(\hat{\theta}, \hat{\chi}\right) W g_{J_s\theta}(\theta_0, \hat{\chi})\right]^{-1} g'_{J_s\theta}\left(\hat{\theta}, \hat{\chi}\right) W$. Expanding $g_{J_s}(\theta_0, \hat{\chi})$ around χ_0,

$$\sqrt{\hat{J}_m}\left(\hat{\theta} - \theta_0\right) = -\Pi\left[\sqrt{\hat{J}_m} g_{J_s}(\theta_0, \chi_0) + \sqrt{\hat{J}_m} g_{J_s\chi}(\theta_0, \chi_0)(\hat{\chi} - \chi_0)\right]. \quad (10)$$

To evaluate equation (10), first note that

$$\begin{aligned}\sqrt{\hat{J}_m} g_{J_s}(\theta_0, \chi_0) &= \sqrt{\hat{J}_m}\left[\bar{m}_{J_m} - m_{J_s}(\theta_0, \chi_0)\right] \\ &= \sqrt{\hat{J}_m}\left[\bar{m}_{J_m} - m(\theta_0, \chi_0)\right] + \sqrt{\hat{J}_m}\left[m(\theta_0, \chi_0) - m_{J_s}(\theta_0, \chi_0)\right]\end{aligned}$$

The two bracketed terms represent independent sets of draws from the same population. The first term equals $\sqrt{\hat{J}_m} g(\theta_0, \chi_0)$, which is asymptotically normally distributed: $\sqrt{\hat{J}_m} g(\theta_0, \chi_0) \rightarrow N(0, \Sigma_g)$. We estimate $\Omega_g = E\left[g(\theta_0, \chi_0) g(\theta_0, \chi_0)'\right]$ directly from its sample counterpart by bootstrapping. The second term represents the simulation error. At the true value of θ, the simulated moments were generated from a finite number of random draws from the true population. Therefore, the second term is also asymptotically normal (as the size of the simulated sample goes to infinity) with mean 0 and variance $\hat{J}_m \frac{\Sigma_g}{J_s}$. Finally, since variation in the simulation and the data are independent, $\sqrt{\hat{J}_m} g_{J_s}(\theta_0, \chi_0) \rightarrow N\left(0, \left(1 + \frac{\hat{J}_m}{J_s}\right)\Sigma_g\right)$. To operationalize this expression for the variance, given the different numbers of observations J_m in the sample, we conservatively use the pairwise maximum numbers of observations, $\max(J_{ma}, J_{mb})$, to weight the (a, b)'th cell of Σ_g in the simulation correction.

Now turn to the second term of equation (10). In the main text we have defined the variance of the first stage parameter estimates $\hat{\chi}$ as $\Omega_\chi = E\left[(\hat{\chi} - \chi_0)(\hat{\chi} - \chi_0)'\right]$.

Thus, $\sqrt{\hat{J}_m} g_{J_s \chi}(\theta_0, \chi_0)(\hat{\chi} - \chi_0) \rightarrow N\left(0, \hat{J}_m G_\chi \Omega_\chi G_\chi'\right)$, and $\sqrt{\hat{J}_m}\left(\hat{\theta} - \theta_0\right) \rightarrow N(0, \Sigma_\theta)$, where equation (10) implies

$$\Sigma_\theta = \left(G_\theta' W G_\theta\right)^{-1} G_\theta' W \left[\left(1 + \frac{\hat{J}_m}{J_s}\right)\Sigma_g + \hat{J}_m \cdot G_\chi \Omega_\chi G_\chi'\right] W G_\theta \left(G_\theta' W G_\theta\right)^{-1}, \qquad (11)$$

by the asymptotic normality of $\hat{\chi}$ and $g(\cdot)$ and by the Slutsky theorem, assuming zero covariance between the first and second stage moments. Dividing by $\hat{J}_m$ we obtain our key equation,

$$\Omega_\theta = Var\left(\hat{\theta}\right) = \left(G'_\theta W G_\theta\right)^{-1} G'_\theta W \left[\Omega_g + \Omega_g^s + G_\chi \Omega_\chi G'_\chi\right] W G_\theta \left(G'_\theta W G_\theta\right)^{-1}.$$

Standard errors reported in the text and tables equal the square roots of the diagonal elements of Ω_θ.

Note that when neither the simulation correction nor the first stage correction matter, we obtain,

$$\Omega_\theta = \left(G'_\theta W G_\theta\right)^{-1} G'_\theta W \Omega_g W G_\theta \left(G'_\theta W G_\theta\right)^{-1}.$$

If we were to assume $W = \Omega_g^{-1}$, this becomes the standard GMM variance formula: $\Omega_\theta = \left(G'_\theta \Omega_g^{-1} G_\theta\right)^{-1}$.

MSM also allows us to perform specification tests. If the model is correct,

$$\begin{aligned} \xi\left(\hat{\theta}, \hat{\chi}\right) &\equiv g'_{J_s}\left(\hat{\theta}, \hat{\chi}\right) \cdot W_{opt} \cdot g_{J_s}\left(\hat{\theta}, \hat{\chi}\right) \\ &= g'_{J_s}\left(\hat{\theta}, \hat{\chi}\right) \cdot \left[\Omega_g + \Omega_g^s + G_\chi \Omega_\chi G'_\chi\right]^{-1} \cdot g_{J_s}\left(\hat{\theta}, \hat{\chi}\right) \end{aligned}$$

will have a chi-squared distribution with $N_m - N_\theta$ degrees of freedom. This test statistic equals $q\left(\hat{\theta}, \hat{\chi}\right)$ in the optimal-weighting case.

The computations in this paper were run on the FASRC cluster supported by the FAS Division of Science Research Computing Group at Harvard University. We start all of our various model estimations from at least twenty different initializations of θ. Though some runs will occasionally time out before converging, we ensure that at least ten different initializations have converged, and then report the point of best fit. Our replication package provides further computational details.

D Identification

D.1 Identification: Description

This section presents additional analysis of our model's identification. Table D1 reports the Andrews et al. (2017) sensitivity measure of $\hat{\theta}$. The numerical derivatives that are used to calculate this sensitivity measure are provided in Table D2. Figure 4 plots the average lifecycle behavior for households in our benchmark $\beta = 1$ model. Figure 5 zooms in on households' illiquid-asset deposit / withdrawal behavior in the benchmark $\hat{\beta} = 0.530$ model. Specifically, Figure 5 splits households based on their income tertile, and shows that illiquid asset accumulation generally occurs in periods of higher income.

D.2 Identification: Results

Table D1: Sensitivity Estimates

	δ	β	ρ
% Visa 21-30	0.022	-0.577	0.996
% Visa 31-40	0.032	-0.816	0.139
% Visa 41-50	-0.007	0.078	-0.879
% Visa 51-60	-0.016	0.290	-1.785
mean Visa 21-30	0.044	-1.050	1.544
mean Visa 31-40	0.023	-0.614	-0.400
mean Visa 41-50	-0.002	-0.028	-1.023
mean Visa 51-60	0.004	-0.140	-1.310
wealth 21-30 | debt	-0.003	0.063	-0.223
wealth 31-40 | debt	-0.004	0.092	-0.591
wealth 41-50 | debt	-0.001	0.038	-0.156
wealth 51-60 | debt	0.003	-0.052	0.258
wealth 21-30 | no debt	-0.003	0.065	-0.586
wealth 31-40 | no debt	0.002	-0.045	-0.056
wealth 41-50 | no debt	0.001	-0.019	-0.117
wealth 51-60 | no debt	0.002	-0.032	-0.079

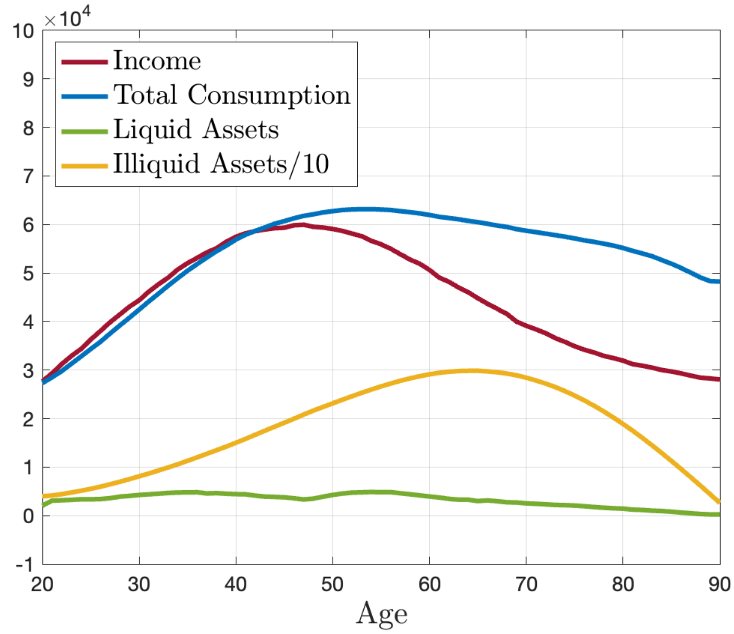
This table reports the sensitivity of $\hat{\theta}$, as defined by Andrews et al. (2017).

Table D2: Numerical Derivatives

	δ	β	ρ
$\Delta m_{J_s}/\Delta\theta$			
% Visa 21-30	-19.292	-0.989	0.006
% Visa 31-40	-24.404	-1.475	-0.039
% Visa 41-50	-28.730	-1.267	-0.005
% Visa 51-60	-27.282	-1.135	-0.045
mean Visa 21-30	-4.519	-0.295	-0.004
mean Visa 31-40	-6.938	-0.401	-0.012
mean Visa 41-50	-8.947	-0.425	-0.008
mean Visa 51-60	-6.857	-0.402	-0.026
wealth 21-30 | debt	34.286	1.808	-0.068
wealth 31-40 | debt	110.140	4.893	-0.548
wealth 41-50 | debt	260.130	11.822	-0.345
wealth 51-60 | debt	548.060	22.110	0.254
wealth 21-30 | no debt	29.115	1.002	-0.347
wealth 31-40 | no debt	108.490	3.665	-0.266
wealth 41-50 | no debt	238.780	8.445	-0.664
wealth 51-60 | no debt	535.430	17.721	-1.529

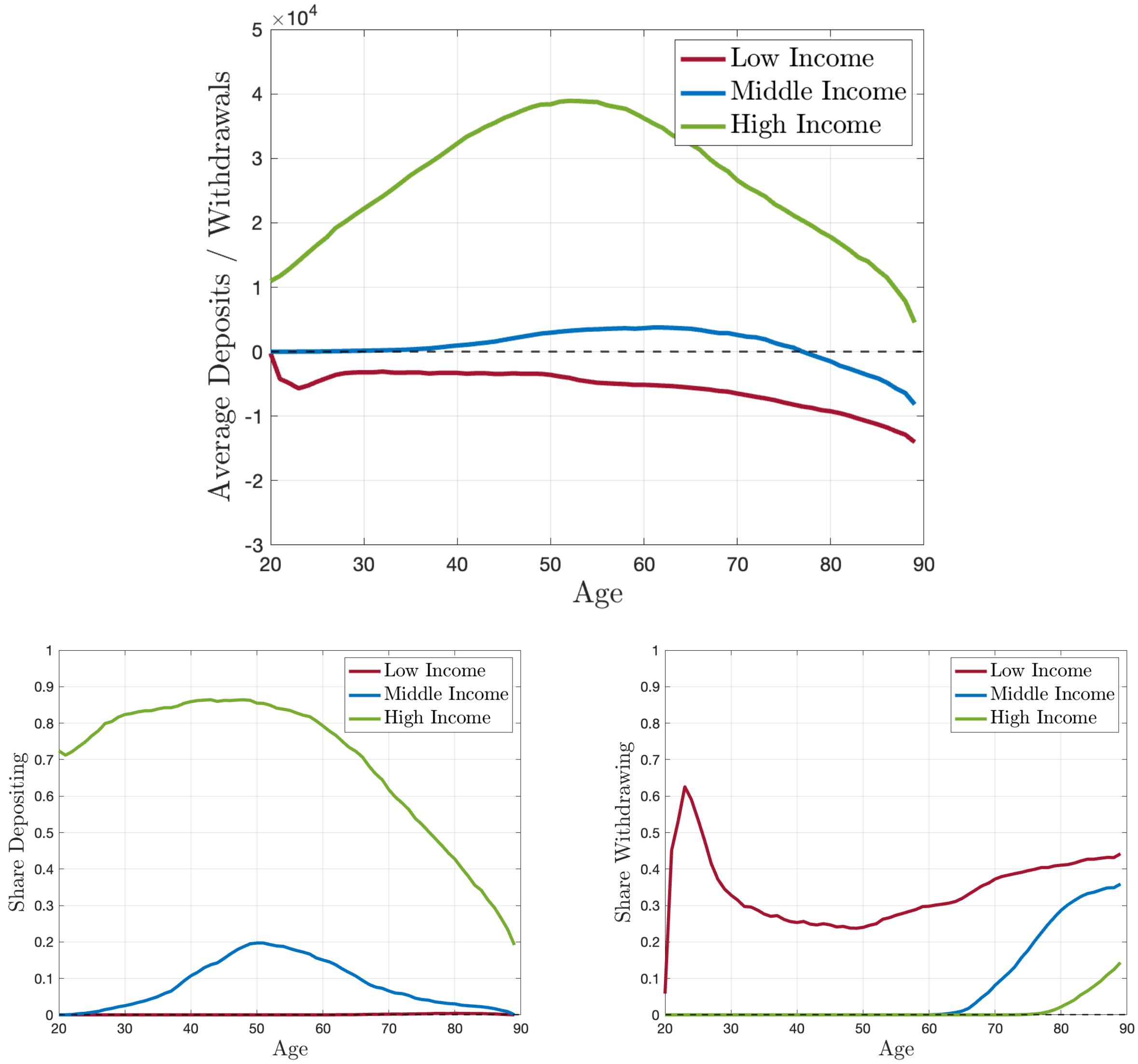
This table reports the model's numerical derivatives. The derivatives in this table are the average of the left-hand and right-hand numerical derivatives, computed at the benchmark parameter estimates reported in Table 3. Derivative step sizes of $d\delta = 0.001$, $d\beta = 0.01$, and $d\rho = 0.01$ are used.

Figure 4: Average Lifecycle Profile for Exponential Estimate



This figure plots the average lifecycle profile of income, total consumption, liquid assets, and illiquid assets (divided by ten for scaling) for the benchmark exponential estimate ($\beta = 1$).

Figure 5: Average Lifecycle Profile of Deposits and Withdrawals



The top panel of this figure plots the lifecycle profile of average net deposits to the illiquid asset for the benchmark estimate ($\hat{\beta} = 0.530$). Similarly, the bottom left (right) panel plots the share of households that deposit (withdraw). Each panel splits households into tertiles of transitory income.

E Robustness

E.1 Robustness: Description

We recognize that economists often disagree on the calibration of ρ. For this reason we examine the effect of imposing several common reasonable values of ρ. Appendix Table E1 reports estimates of the present-biased discount function conditional on $\rho \in \{0.5, 1, 2, 3\}$.

Table E2 examines robustness to our return assumptions, since R, R^{CC}, and γ are not included in the first-stage correction. Columns 2 and 3 perturb R to 1.015 and 1.025, respectively. In Columns 4 and 5 we assume $R^{CC} = 1.09$ and $R^{CC} = 1.12$. In Column 6 we set $\gamma = 3.38\%$, corresponding to the average tax- and inflation-adjusted mortgage interest rate from 1980-2000, as calculated from Freddie Mac's historical series of nominal mortgage interest rates and the CPI-U, assuming a marginal tax rate of 25%.[54] In Column 7, we consider the case $\gamma = 6.59\%$. Flavin and Yamashita (2002) calculate this as the average real after-tax return to housing, including capital gains, use-value, maintenance costs, and taxes.

Appendix Table E3 reports the results of modifying various aspects of the income process, including adjusting the persistence of income shocks and their dynamics. Details of these robustness checks are included in the table's footnote.

Table E4 looks at some of the model's remaining structural assumptions. In Column 2 we completely restrict withdrawals from the illiquid account. Column 3 studies alternate assumptions about returns to scale in household size. Columns 4 and 5 increase and decrease the strength of the bequest motive (α). Column 6 examines alternate assumptions about the household death process.[55] Column 7 assumes that all simulated households begin with exactly zero initial wealth.

[54] This choice for γ reflects the interest savings resulting from paying off a dollar of mortgage debt.

[55] In the benchmark model both heads of house are assumed to die simultaneously. In Column 6 we incorporate individual mortality. We assume that utility depends continuously on the expected number of heads that remain alive until age 84, at which point the expected number of surviving heads falls below 1. After age 84 we retain our original assumption that mortality is a discrete event.

Robustness to the weighting matrix is examined in Table E5. We re-estimate our results using the identity matrix, the inverse of the full VCV matrix $\hat{\Omega}_g$, and the optimal weighting matrix.

Table E6 examines robustness to joint assumptions on R, R^{CC}, and γ. As discussed in Section 6.3, we estimate our model using the calibration of Kaplan and Violante (2014). The first column of Table E6 estimates the model under our baseline calibration, but sets the bequest motive to 0 (in line with Kaplan and Violante (2014)). Columns 2, 3, and 4 report estimates of $\hat{\theta}$ under interest rate calibrations that are a weighted average of our benchmark calibration ($R = 1.0203, R^{CC} = 1.1059, \gamma = 0.05$) and the Kaplan and Violante (2014) calibration ($R = 0.9852, R^{CC} = 1.06, \gamma = 0.0629$). Columns 5 and 6 estimate $\hat{\theta}$ under the Kaplan and Violante (2014) calibration, first without and then with the restriction that $\beta = 1$. The wedge between R^{CC} and γ shrinks as we move from Column 1 to Column 5. As detailed in the main text, estimates of $\hat{\beta}$ increase accordingly.

A similar analysis is also shown graphically in Figure 6, which provides a boundary analysis in (β, δ)-space (fixing $\rho = 2$) for the interest rate calibrations explored in Table E6. The key takeaway from that analysis is that, as the wedge between R^{CC} and γ declines, the red credit card boundary shifts rightward toward higher values of β.

Finally, Table E7 provides a preliminary analysis of household return expectations. Specifically, we fix $\beta = 1$ and allow for households' perceived return on the illiquid asset to increase. As households' perceived return approaches the credit card borrowing rate, the $\beta = 1$ model's fit of the data improves.

E.2 Robustness: Results

Table E1: Risk Aversion

	(1) $\rho = 0.5$	(2) $\rho = 1$	(3) $\rho = 2$	(4) $\rho = 3$
Present Biased				
Parameter estimates				
$\hat{\beta}$	0.2593	0.3088	0.5302	0.3913
s.e. (i)	(0.0198)	(0.0466)	(0.1103)	(0.0627)
$\hat{\delta}$	0.9962	0.9953	0.9891	0.9948
s.e. (i)	(0.0005)	(0.0013)	(0.0058)	(0.0027)
$\hat{\rho}$	0.5	1	2	3
s.e. (i)	-	-	-	-
Goodness-of-fit				
$q(\hat{\theta}, \hat{\chi})$	132.68	91.60	77.39	85.47
Exponential				
Parameter estimates				
$\hat{\beta}$	1	1	1	1
s.e. (i)	-	-	-	-
$\hat{\delta}$	0.9555	0.9564	0.9657	0.9700
s.e. (i)	(0.0019)	(0.0033)	(0.0058)	(0.0072)
$\hat{\rho}$	0.5	1	2	3
s.e. (i)	-	-	-	-
Goodness-of-fit				
$q(\hat{\theta}, \hat{\chi})$	1222.80	793.92	786.06	891.99

This table reports estimates of the discount function, conditional on different ρ values. Columns (1) through (4) fix only ρ, maintaining all other benchmark assumptions.

Table E2: Rates of Return

	(1) Benchmark	(2) $r = 1.5\%$	(3) $r = 2.5\%$	(4) $r^{CC} = 9\%$	(5) $r^{CC} = 12\%$	(6) $\gamma = 3.38\%$	(7) $\gamma = 6.59\%$
Present Biased							
Parameter estimates							
$\hat{\beta}$	0.5305	0.4885	0.5739	0.4995	0.5414	0.3670	0.5564
s.e. (i)	(0.1143)	(0.1073)	(0.0977)	(0.0849)	(0.0983)	(0.0843)	(0.0789)
$\hat{\delta}$	0.9891	0.9916	0.9861	0.9910	0.9881	0.9948	0.9840
s.e. (i)	(0.0051)	(0.0034)	(0.0048)	(0.0032)	(0.0048)	(0.0022)	(0.0046)
$\hat{\rho}$	1.9355	2.3641	1.6011	2.3842	1.6905	0.7500	2.4071
s.e. (i)	(0.4350)	(0.8428)	(0.3567)	(0.8505)	(0.4385)	(0.1057)	(0.6177)
Goodness-of-fit							
$q(\hat{\theta}, \hat{\chi})$	77.15	64.20	92.72	55.38	101.06	136.65	48.69
Exponential							
Parameter estimates							
$\hat{\beta}$	1	1	1	1	1	1	1
s.e. (i)	-	-	-	-	-	-	-
$\hat{\delta}$	0.9601	0.9586	0.9597	0.9589	0.9614	0.9812	0.9482
s.e. (i)	(0.0053)	(0.0054)	(0.0048)	(0.0045)	(0.0057)	(0.0023)	(0.0049)
$\hat{\rho}$	1.4663	1.5197	1.3699	1.2490	1.7246	1.4342	1.4231
s.e. (i)	(0.2256)	(0.3375)	(0.3314)	(0.1876)	(0.6420)	(0.3332)	(0.3886)
Goodness-of-fit							
$q(\hat{\theta}, \hat{\chi})$	759.57	686.47	836.94	592.54	901.50	1329.60	409.60

This table studies robustness to the model's return assumptions. Columns (2) through (7) perturb the model's three interest rates one at a time. The benchmark assumes that $r = 2.03\%$, $r^{CC} = 10.59\%$, and $\gamma = 5\%$ (see Table 2).

Table E3: Income Process

	(1) Benchmark	(2) Split Income at Retirement	(3) Income Variance × 2	(4) $\psi = 0.9$	(5) $\psi = 0.9$ Full Update
Present Biased					
Parameter estimates					
$\hat{\beta}$	0.5305	0.4877	0.2115	0.4860	0.5770
s.e. (i)	(0.1143)	(0.0867)	(0.0466)	(0.0750)	(0.1319)
$\hat{\delta}$	0.9891	0.9906	0.9965	0.9867	0.9839
s.e. (i)	(0.0051)	(0.0031)	(0.0012)	(0.0090)	(0.0039)
$\hat{\rho}$	1.9355	2.0698	3.7217	2.7306	2.1821
s.e. (i)	(0.4350)	(1.7831)	(0.2146)	(0.3297)	(0.6817)
Goodness-of-fit					
$q(\hat{\theta}, \hat{\chi})$	77.15	85.56	90.19	91.53	100.71
Exponential					
Parameter estimates					
$\hat{\beta}$	1	1	1	1	1
s.e. (i)	-	-	-	-	-
$\hat{\delta}$	0.9601	0.9588	0.9528	0.9502	0.9504
s.e. (i)	(0.0053)	(0.0051)	(0.0036)	(0.0102)	(0.0053)
$\hat{\rho}$	1.4663	1.3899	0.8098	1.4442	1.4900
s.e. (i)	(0.2256)	(0.1883)	(0.1358)	(0.7516)	(0.1521)
Goodness-of-fit					
$q(\hat{\theta}, \hat{\chi})$	759.57	762.71	679.55	733.98	732.63

This table reports estimates of the discount function under different income process assumptions. Column (2) uses an income process that is separately estimated over ages 20-63 and 64-90. Column (3) reports the effect of doubling σ_ϵ^2 and σ_ν^2, which are the variances of the i.i.d. terms in the income process (see Table 2). The variable ψ is the autocorrelation coefficient in the income process, and measures the persistence of shocks. Column (4) reports the effect of increasing ψ to 0.9. Column (5) also updates estimates of the other income parameters.

Table E4: Other Structural Assumptions

	(1) Benchmark	(2) Completely Illiquid Z Account	(3) Square Root Scale	(4) Bequest × 1.25	(5) Bequest × 0.75	(6) Partial Individual Mortality	(7) Zero Initial Wealth
Present Biased							
Parameter estimates							
$\hat{\beta}$	0.5305	0.2970	0.5226	0.5998	0.5073	0.5158	0.5505
s.e. (i)	(0.1143)	(0.0437)	(0.0766)	(0.1134)	(0.1069)	(0.0693)	(0.0808)
$\hat{\delta}$	0.9891	0.9957	0.9917	0.9841	0.9916	0.9931	0.9884
s.e. (i)	(0.0051)	(0.0026)	(0.0028)	(0.0061)	(0.0044)	(0.0019)	(0.0030)
$\hat{\rho}$	1.9355	3.8357	2.0985	1.5866	2.0643	2.1246	1.3513
s.e. (i)	(0.4350)	(0.1349)	(0.4073)	(0.3802)	(0.5226)	(0.6315)	(0.4279)
Goodness-of-fit							
$q(\hat{\theta}, \hat{\chi})$	77.15	38.75	97.95	77.34	77.08	79.52	157.69
Exponential							
Parameter estimates							
$\hat{\beta}$	1	1	1	1	1	1	1
s.e. (i)	-	-	-	-	-	-	-
$\hat{\delta}$	0.9601	0.9592	0.9700	0.9634	0.9589	0.9657	0.9708
s.e. (i)	(0.0053)	(0.0044)	(0.0050)	(0.0047)	(0.0046)	(0.0055)	(0.0039)
$\hat{\rho}$	1.4663	1.1051	1.8165	1.6302	1.3038	1.4349	1.3535
s.e. (i)	(0.2256)	(0.3077)	(0.3460)	(0.2841)	(0.3103)	(0.3502)	(0.3605)
Goodness-of-fit							
$q(\hat{\theta}, \hat{\chi})$	759.57	861.19	824.94	752.67	763.41	816.59	949.24

This table studies robustness to additional structural assumptions. Columns (2) through (7) perturb the model's assumptions on the illiquid account (Z), returns to scale within the household, the bequest motive, and model initialization. The completely illiquid Z account condition disallows any withdrawals from Z. The square root scale assumes that, if the household size is n, period utility equals $n \cdot u(\frac{c}{\sqrt{n}})$, where c is total consumption. Columns (4) and (5) respectively increase and decrease the strength of the bequest motive by 25%. "Partial Individual Mortality" puts mortality into n, so that n falls in proportion to unconditional survival probabilities. Column (7) assumes that all simulated households begin with exactly zero initial wealth.

Table E5: Weighting Matrix

	(1) Baseline	(2) Identity Matrix	(3) Full VCV	(4) Optimal Weights with Adjustments
Present Biased				
Parameter estimates				
$\hat{\beta}$	0.5305	0.5427	0.2954	0.1741
s.e. (i)	(0.1143)	(0.1228)	(0.0401)	(0.0118)
$\hat{\delta}$	0.9891	0.9888	0.9953	0.9977
s.e. (i)	(0.0051)	(0.0048)	(0.0012)	(0.0003)
$\hat{\rho}$	1.9355	1.7812	0.7831	0.8070
s.e. (i)	(0.4350)	(0.5090)	(0.1209)	(0.1508)
Goodness-of-fit				
$q(\hat{\theta}, \hat{\chi})$	77.15	1.60	216.02	56.29
Exponential				
Parameter estimates				
$\hat{\beta}$	1	1	1	1
s.e. (i)	-	-	-	-
$\hat{\delta}$	0.9601	0.9669	0.9214	0.9218
s.e. (i)	(0.0053)	(0.0041)	(0.0094)	(0.0025)
$\hat{\rho}$	1.4663	1.1857	1.1896	0.5110
s.e. (i)	(0.2256)	(0.6422)	(0.2095)	(0.1254)
Goodness-of-fit				
$q(\hat{\theta}, \hat{\chi})$	759.57	2.97	458.70	158.95

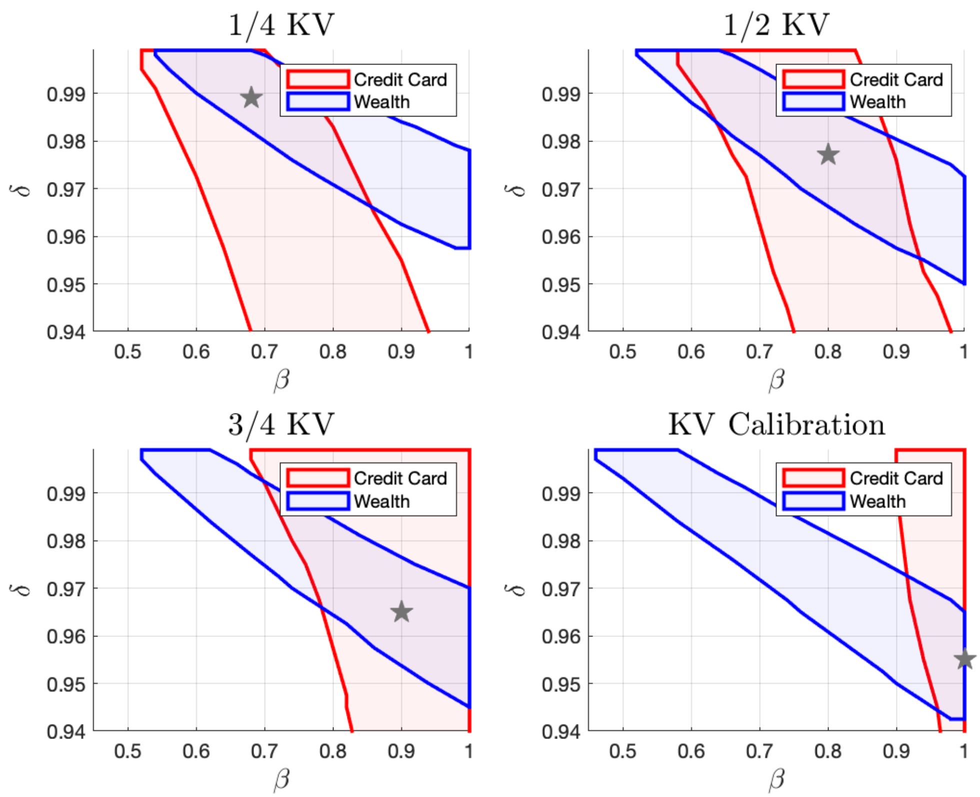
This table reports estimates of the discount function under alternate weighting matrices, as indicated in the column headers.

Table E6: Kaplan and Violante (2014) Calibration

	(1) Benchmark (w/o bequest motive)	(2) 3/4 Benchmark 1/4 KV	(3) 1/2 Benchmark 1/2 KV	(4) 1/4 Benchmark 3/4 KV	(5) KV Calibration	(6) KV Calibration Exponential	(7) Data
Parameter estimates							
$\hat{\beta}$	0.5851	0.5948	0.7838	0.9381	1.0436	1	-
s.e. (i)	(0.1600)	(0.0880)	(0.0631)	(0.0751)	(0.0376)	-	-
$\hat{\delta}$	1.0000	0.9995	0.9781	0.9622	0.9500	0.9550	-
s.e. (i)	(0.0223)	(0.0151)	(0.0122)	(0.0109)	(0.0071)	(0.0069)	-
$\hat{\rho}$	2.0625	2.5123	2.0424	1.6075	1.2969	1.9614	-
s.e. (i)	(0.6131)	(0.5320)	(0.2188)	(0.7250)	(0.2923)	(0.8068)	-
Second-stage moments							
% Visa 21-30	0.511	0.531	0.533	0.528	0.516	0.528	0.640
% Visa 31-40	0.538	0.562	0.584	0.590	0.595	0.602	0.629
% Visa 41-50	0.565	0.590	0.623	0.646	0.666	0.685	0.588
% Visa 51-60	0.585	0.638	0.674	0.704	0.743	0.767	0.503
mean Visa 21-30	0.081	0.081	0.078	0.073	0.067	0.072	0.111
mean Visa 31-40	0.103	0.104	0.104	0.100	0.096	0.101	0.096
mean Visa 41-50	0.133	0.133	0.137	0.137	0.138	0.144	0.110
mean Visa 51-60	0.136	0.160	0.184	0.193	0.204	0.208	0.104
wealth 21-30 | debt	1.028	1.163	1.290	1.427	1.512	1.431	1.222
wealth 31-40 | debt	1.674	1.930	2.060	2.215	2.312	2.200	1.868
wealth 41-50 | debt	2.830	3.152	3.291	3.429	3.454	3.405	3.377
wealth 51-60 | debt	4.228	4.739	4.858	5.021	5.014	5.020	4.650
wealth 21-30 | no debt	2.319	2.231	2.149	2.086	2.008	2.030	1.659
wealth 31-40 | no debt	3.199	3.071	3.002	3.005	2.937	2.929	2.800
wealth 41-50 | no debt	4.095	4.074	4.031	4.154	4.221	4.133	4.613
wealth 51-60 | no debt	5.265	5.554	5.712	5.975	6.219	6.326	8.071
Goodness-of-fit							
$q(\hat{\theta}, \hat{\chi})$	164.15	131.36	135.83	147.52	171.72	172.11	-

This table compares our model to Kaplan and Violante (2014). The estimates in Column (1) maintain all benchmark assumptions except that the bequest motive is removed to align with the Kaplan and Violante (2014) model. The estimates in Columns (5) and (6) are based on the return assumptions employed by Kaplan and Violante (2014): $\gamma = 6.29\%$, $r = -1.48\%$, and $r^{CC} = 6\%$. In Columns (2), (3), and (4), the estimates are based on linear combinations of our benchmark return assumptions and those of Kaplan and Violante (2014).

Figure 6: Interest Rate Boundary Analysis



This figure shows a boundary analysis in (β, δ)-space (fixing $\rho = 2$) for the interest rate calibrations explored in Table E6. See Section 6.1 for boundary analysis details.

Table E7: Perceived Return on Illiquid Wealth

	(1) Benchmark	(2) 7% Perceived Return	(3) 10% Perceived Return	(4) Data
Parameter estimates				
$\hat{\beta}$	1	1	1	-
s.e. (i)	-	-	-	-
$\hat{\delta}$	0.9601	0.9518	0.9712	-
s.e. (i)	(0.0053)	(0.0063)	(0.0095)	-
$\hat{\rho}$	1.4663	1.3823	3.9326	-
s.e. (i)	(0.2256)	(0.2800)	(0.0603)	-
Second-stage moments				
% Visa 21-30	0.309	0.410	0.484	0.640
% Visa 31-40	0.287	0.474	0.551	0.629
% Visa 41-50	0.299	0.535	0.604	0.588
% Visa 51-60	0.257	0.583	0.638	0.503
mean Visa 21-30	0.044	0.054	0.065	0.111
mean Visa 31-40	0.051	0.078	0.093	0.096
mean Visa 41-50	0.059	0.112	0.130	0.110
mean Visa 51-60	0.034	0.138	0.157	0.104
wealth 21-30 | debt	0.880	1.351	1.624	1.222
wealth 31-40 | debt	0.999	2.201	2.476	1.868
wealth 41-50 | debt	1.941	3.369	3.478	3.377
wealth 51-60 | debt	3.811	4.834	5.180	4.650
wealth 21-30 | no debt	2.017	2.097	2.094	1.659
wealth 31-40 | no debt	2.850	2.942	2.984	2.800
wealth 41-50 | no debt	3.967	4.005	4.359	4.613
wealth 51-60 | no debt	5.358	5.649	6.943	8.071
Goodness-of-fit				
$q(\hat{\theta}, \hat{\chi})$	759.57	245.66	147.61	-

This table reports estimates of the (exponential) discount function under alternative assumptions about households' perceived return on the illiquid asset (holding the actual return at 5%).

F Extensions

F.1 Extensions: Description

Table F1 presents the heterogeneity analysis described in Section 7.1. Table F2 presents our estimates under the alternate assumption that households are sophisticated about their self-control problems. Table F3 estimates θ for alternate education groups.

F.2 Extensions: Results

Table F1: Discount Factor Heterogeneity in Exponential Model

	(1) $\rho = 0.5$	(2) $\rho = 1$	(3) $\rho = 2$	(4) $\rho = 3$	(5) Data
Parameter estimates					
$\widehat{\mathbb{E}(\delta)}$	0.9235	0.9194	0.8702	0.7930	-
Second-stage moments					
% Visa 21-30	0.445	0.457	0.464	0.458	0.640
% Visa 31-40	0.483	0.468	0.460	0.460	0.629
% Visa 41-50	0.486	0.468	0.467	0.474	0.588
% Visa 51-60	0.456	0.437	0.447	0.467	0.503
mean Visa 21-30	0.066	0.068	0.069	0.071	0.111
mean Visa 31-40	0.091	0.087	0.088	0.091	0.096
mean Visa 41-50	0.114	0.108	0.112	0.118	0.110
mean Visa 51-60	0.141	0.125	0.132	0.146	0.104
wealth 21-30 | debt	0.614	0.642	0.509	0.367	1.222
wealth 31-40 | debt	0.615	0.746	0.728	0.611	1.868
wealth 41-50 | debt	0.923	1.215	1.500	1.483	3.377
wealth 51-60 | debt	1.409	1.950	2.359	2.723	4.650
wealth 21-30 | no debt	2.020	2.039	2.016	2.017	1.659
wealth 31-40 | no debt	3.451	3.305	3.187	3.201	2.800
wealth 41-50 | no debt	5.183	5.184	5.148	5.150	4.613
wealth 51-60 | no debt	7.339	7.925	8.905	9.076	8.071
Goodness-of-fit					
$q(\hat{F}, \hat{\chi})$	470.68	393.51	364.27	383.86	-

This table allows for δ heterogeneity in the Exponential case. Columns (1) through (4) explore different ρ values. Details are provided in Section 7.1. For reference, Column (5) lists the empirical moments. When $\rho = 2$ (the best-fitting case), the estimated parameters of the beta distribution are $\alpha = 3.79$ and $\beta = 0.56$. We do not calculate standard errors on the parameters of the distribution.

Table F2: Sophisticated Present Bias

	(1) $\rho = 0.5$	(2) $\rho = 1$	(3) $\rho = 2$	(4) $\rho = 3$	(5) Data
Parameter estimates					
$\hat{\beta}$	0.8519	0.7817	0.5670	0.4174	-
s.e. (i)	(0.0139)	(0.0239)	(0.1185)	(0.0553)	-
$\hat{\delta}$	0.9678	0.9746	0.9867	0.9916	-
s.e. (i)	(0.0020)	(0.0031)	(0.0058)	(0.0039)	-
$\hat{\rho}$	0.5	1	2	3	-
s.e. (i)	-	-	-	-	-
Second-stage moments					
% Visa 21-30	0.553	0.586	0.642	0.646	0.640
% Visa 31-40	0.584	0.592	0.611	0.595	0.629
% Visa 41-50	0.602	0.594	0.549	0.514	0.588
% Visa 51-60	0.567	0.579	0.481	0.436	0.503
mean Visa 21-30	0.094	0.096	0.108	0.107	0.111
mean Visa 31-40	0.120	0.116	0.122	0.119	0.096
mean Visa 41-50	0.139	0.136	0.125	0.117	0.110
mean Visa 51-60	0.104	0.119	0.109	0.096	0.104
wealth 21-30 | debt	1.229	1.247	1.000	0.870	1.222
wealth 31-40 | debt	1.878	1.950	1.638	1.387	1.868
wealth 41-50 | debt	2.819	3.076	2.963	2.657	3.377
wealth 51-60 | debt	3.853	4.574	5.235	5.126	4.650
wealth 21-30 | no debt	2.365	2.379	2.296	2.175	1.659
wealth 31-40 | no debt	3.205	3.254	3.163	2.962	2.800
wealth 41-50 | no debt	3.904	4.237	4.564	4.377	4.613
wealth 51-60 | no debt	4.688	5.651	7.166	7.143	8.071
Goodness-of-fit					
$q(\hat{\theta}, \hat{\chi})$	161.00	101.21	54.03	67.71	-

This table extends our estimates to sophisticated present bias. Each column fixes ρ, and reports conditional estimates of β and δ.

Table F3: Education

	(1) Present Biased Some H.S.	(2) Present Biased Comp. H.S.	(3) Present Biased Comp. College	(4) Data		
Parameter estimates						
$\hat{\beta}$	0.6860	0.5305	0.7914		-	
s.e. (i)	(0.1595)	(0.1143)	(0.2177)		-	
$\hat{\delta}$	0.9789	0.9891	0.9835		-	
s.e. (i)	(0.0092)	(0.0051)	(0.0063)		-	
$\hat{\rho}$	1.2080	1.9355	0.8992		-	
s.e. (i)	(0.6002)	(0.4350)	(0.1622)		-	
Second-stage moments				Some H.S.	Comp H.S.	College
% Visa 21-30	0.458	0.605	0.437	0.462	0.640	0.509
% Visa 31-40	0.513	0.585	0.303	0.528	0.629	0.343
% Visa 41-50	0.591	0.523	0.295	0.596	0.588	0.329
% Visa 51-60	0.664	0.475	0.263	0.534	0.503	0.217
mean Visa 21-30	0.044	0.103	0.097	0.081	0.111	0.104
mean Visa 31-40	0.080	0.117	0.043	0.091	0.096	0.070
mean Visa 41-50	0.141	0.124	0.030	0.162	0.110	0.079
mean Visa 51-60	0.189	0.116	0.028	0.189	0.104	0.077
wealth 21-30 | debt	1.099	0.913	2.074	1.185	1.222	2.338
wealth 31-40 | debt	2.036	1.412	2.771	2.355	1.868	3.180
wealth 41-50 | debt	3.347	2.640	5.117	3.802	3.377	5.240
wealth 51-60 | debt	4.592	4.723	9.627	4.882	4.650	7.568
wealth 21-30 | no debt	2.189	2.324	2.949	2.094	1.659	3.989
wealth 31-40 | no debt	3.511	3.248	3.610	2.908	2.800	4.708
wealth 41-50 | no debt	4.782	4.633	5.743	4.836	4.613	8.250
wealth 51-60 | no debt	6.183	7.475	9.962	5.938	8.071	13.681
Goodness-of-fit						
$q(\hat{\theta}, \hat{\chi})$	3.45	77.15	96.46		-	

This table reports estimates for households whose heads have varying levels of education. Column (2) copies our benchmark estimates, which correspond to the "High School" education group. Column (1) pertains to households whose heads did not receive a high school degree, while Column (3) pertains to college graduates.